SCARBOROUGH CAMPUS
UNIVERSITY OF TORONTO

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<u>Term Test I</u>

1. Consider the rational function $f(x) = \dfrac{(4x^2 - 4x + 1)(x+1)}{(2x+1)(x^2 + x - 2)}$.

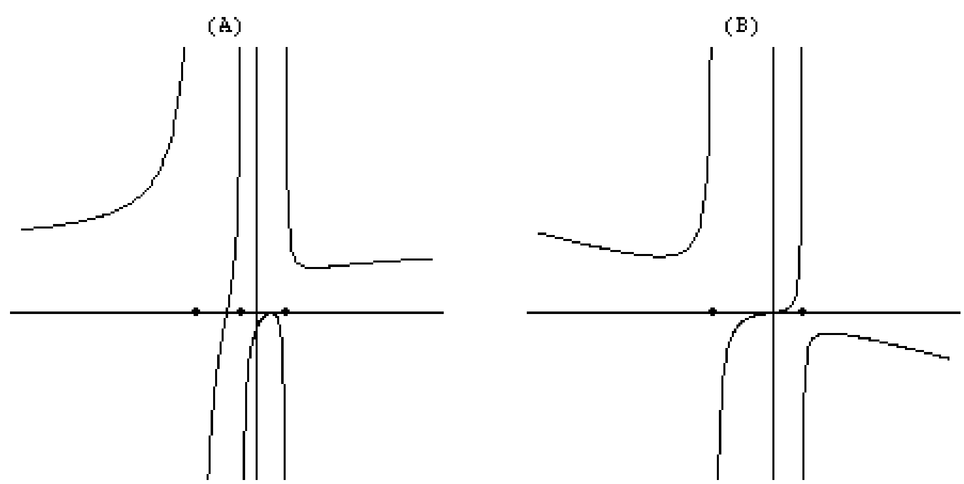
[**4**] (a) What is the domain of f?

[**4**] (b) What are the zeros of f?

[**3**] (c) What is the the order of f?

[**8**] (d) Find those intervals on which $f(x) < 0$.

[**8**] (e) One of the graphs (A) or (B) below depicts the graph of $f(x)$, where a $\bullet$ denotes an argument at which the function is not defined. Using the results in (a) through (d), state at least two reasons why one graph is correct and why the other one is not.



2. Let $f(x) = \frac{x^4}{2} - 3x^2 + 5$.

[**5**] (a) Find the linear approximation $f_1(x)$ for $f(x)$ at $a = 1$.

[**2**] (b) Write the error function $E_1(x)$ associated to $f_1(x)$.

[**8**] (c) Find a positive number h such that $|f(x) - f_1(x)| \leq \frac{1}{100}$ when $x \in [1-h, 1+h]$. (*Hint:* Try $h_0 = 1$ as your initial guess.)

[4] 3. (a) State the hypotheses that allow you to apply the special case of Newton's method to a polynomial $P(x)$ on an interval $[a, b]$.

[4] (b) Prove that there is exactly one root r of $P(x) = x^5 - 4x^2 + 1$ in $[0.5, 0.6]$ and that the special case of Newton's method applies.

[8] (c) Show that on the interval $[0.5, 1]$ one has $|P'(x)| \geq 3$ and $|P''(x)| \leq 12$.

[12] (d) Apply the special case of Newton's method to find the root r from (b) to within 10^{-8}.

4. Solve the following inequalities:

[5] (a) $|x - 3| > 3x + 2$.

[5] (b) $\dfrac{|x|}{(x^2 - 1)} > 0$.

[5] (c) $\dfrac{(x - 3)}{(x - 1)} > x - 4$.

[4] 5. (a) State the Mean Value Theorem (MVT) for rational functions. Clearly indicate what are the hypotheses and what is the conclusion.

[4] (b) Prove that MVT applies to $f(x) = \frac{x}{(1+x^2)}$ on $[0, 10]$ and state its conclusion in this situation.

[7] (c) For how many points $z \in (0, 10)$ does the conclusion of MVT hold for the function f and the interval given in (b)? (You need not determine the numerical values of those z.)