

**SCARBOROUGH CAMPUS
UNIVERSITY OF TORONTO**

MATA26Y

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Term Test II

- [25] 1. (a) State the Chain Rule.
(b) Find the derivatives of the following functions.
- i) $f(x) = \cos(x + e^x)$
 - ii) $f(x) = (x^3 + 3x) \arctan x$
 - iii) $f(x) = x^{\ln x}, \quad x > 0$
 - iv) $f(x) = \frac{(2 + \sin x)^{\frac{1}{3}}}{\sqrt{1 + x^2}}$

- [30] 2. Let $f(x) = \frac{18(x-1)}{x^2}$. Find
- (a) the domain of f ,
 - (b) the zeros of f ,
 - (c) the intervals on which f is positive/negative,
 - (d) the intervals of increase/decrease,
 - (e) the minima and maxima,
 - (f) the intervals on which f is concave up/down,
 - (g) the inflection points,
 - (h) the vertical asymptotes, and
 - (i) the horizontal asymptotes.

Sketch the graph of f on the **next page**, and indicate all of the above information on the graph.

$$\left(\text{Hint: } f'(x) = \frac{18(-x+2)}{x^3} \text{ and } f''(x) = \frac{36(x-3)}{x^4} \right)$$

- [20] 3. Find the following limits.
- (a) $\lim_{x \rightarrow \pi} \frac{\cos^{25}(x)}{\sqrt{x}}$
 - (b) $\lim_{x \rightarrow 3} \frac{x^3 - 5x^2 + 3x + 9}{x^2 - 6x + 9}$
 - (c) $\lim_{x \rightarrow 1} \sin\left(\frac{1}{x-1}\right) \ln(x^2)$
 - (d) $\lim_{x \rightarrow 0^+} (\sin x \ln x)$

- [8] 4. Find the equation of the tangent line to the graph of $f(x) = x \cos x$ at $\left(\frac{\pi}{6}, \frac{\pi\sqrt{3}}{12}\right)$.
- [5] 5. Solve for x : $\ln(x^2 - 2x - 3) - \ln(x + 1) = 0$.
- [12] 6. (a) Use implicit differentiation to find y' for the equation $xy^5 + x^5y = 1$.
(b) Show that the graph of $xy^5 + x^5y = 1$ has no horizontal tangents.