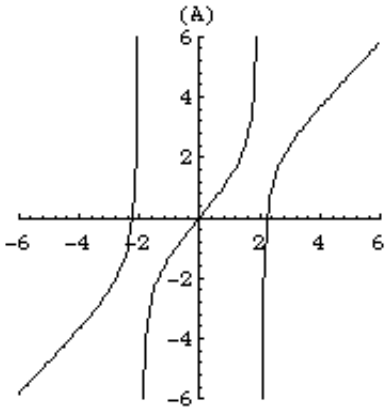
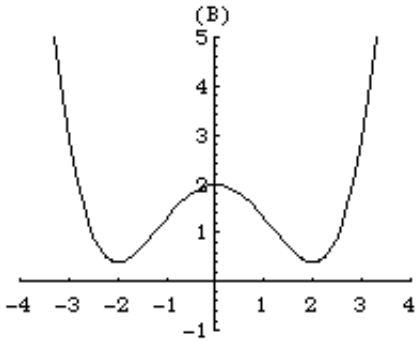
**UNIVERSITY OF TORONTO**
**SCARBOROUGH CAMPUS**

MATA26Y October 30, 1995

TERM TEST I

1. (a) Match the following statements with one (or both) of the pictures below, by checking the appropriate column(s).




[**1**] (i) This rational function has 3 zeros but its derivative has no zero.

[**1**] (ii) This function has no zero, but its derivative has 3 zeros.

[**1**] (iii) Neither this function nor its derivative take on a maximum value on the interval $(-1, 1)$.

[**1**] (iv) On $[-1, 1]$, the derivative takes on its minimum inside the interval, but its maximum is taken on at an endpoint.

[**1**] (v) There is no horizontal tangent.

[**1**] (vi) There is a tangent that passes through the graph of the curve at the point of tangency (an inflection point).

[**1**] (vii) The slope of any tangent line is positive.

| | (A) | (B) |
|---|---|---|
| (i) | | |
| (ii) | | |
| (iii) | | |
| (iv) | | |
| (v) | | |
| (vi) | | |
| (vii) | | |

[**4**] (b) (i) State Rolle's Theorem for rational functions. Clearly indicate what are the hypotheses and what is the conclusion.

[**1**] (ii) How do you console your answer to (a)(i) with Rolle's Theorem.

**[8]** 1. (c) Find possible equations for the functions depicted.

2. Let $f(x) = x^3 - 3x - 3$.

**[5]** (a) Does the special case of Newton's method apply to $f(x)$ on $[2, 4]$? Explain.

**[9]** (b) Determine appropriate constants $K$ and $h$ and subinterval $[u, v] \subset [2, 4]$ so that Newton's method is guaranteed to work.

**[6]** (c) Determine the number of iterations necessary to approximate the root to within $10^{-6}$ and do the necessary calculations to find the approximation.
(If you used a program on a calculator, please state this fact.)

3. Let $f(x) = \dfrac{1}{x^2 - 9}$.

**[6]** (a) Find the constant approximation $A_0(x)$ and the linear approximation $A_1(x)$ for $f(x)$ at $a = 5$. Then find the error functions $E_0(x)$ and $E_1(x)$.

**[9]** (b) Find upper bounds for the absolute values $|E_0(x)|$ and $|E_1(x)|$ in the interval $|a - x| \leq 1$.

4. Let $g(x) = \dfrac{1}{x^2 - 2x}$.

**[6]** (a) Write down the equations of the tangent lines to the graph of $g(x)$ at $a = 0.5,\ 1,\ 1.5$ respectively.

**[6]** (b) Prove that on $\left[\frac{1}{2}, \frac{3}{2}\right]$ the graph of $g(x)$ lies above each of the three tangent lines in (a) and sketch the graph with its three tangent lines.
$\left(\textit{Hint:}\quad g''(x) = 2\dfrac{3x^2 - 4x + 2}{(x^2 - 2x)^3}.\right)$

**[3]** (c) Find $u, v \in \left[\frac{1}{2}, \frac{3}{2}\right]$ such that $u < v$ and the tangent line to the graph of $g(x)$ at $a = 1$ is a better approximation to the graph over $(u, v)$ than both the tangent lines at $a = 0.5$ or at $a = 1.5$.

5. Let $f(x) = |x^2 - 5x + 6| - |x|$.

**[6]** (a) Find all solutions of $f(x) = 0$.

**[4]** (b) Determine where $f(x) < 0$.

6. The study of resonance leads to the family of functions

$$y_a(x) = \frac{1}{(1 - x^2)^2 + 2ax^2} \quad , \quad x \geq 0, \quad a > 0\ .$$

**[2]** (a) Show that $y_a(x) < \dfrac{1}{(1 - x^2)^2}$ for $x > 0$.

**[8]** (b) For which values of $a$ is it true that the function $y_a(x)$ achieves its maximal value on $[0, 1]$ at
- the left endpoint?
- the right endpoint?
- neither of the endpoints?

**[5]** (c) For which values of $a$ is it true that the graph of the function $y_a(x)$ over $[0, 1]$ does not
- extend below the line $y = 1$?
- extend above the line $y = 1$?

**[5]** (d) Sketch the graphs for $a = 0.05, 0.10, 1.00, 3.00$ and $x > 0$ on one set of axes, together with $y(x) = \dfrac{1}{(1 - x^2)^2}$, the graph for $a = 0$. You may check your work using the calculator!