

UNIVERSITY OF TORONTO
SCARBOROUGH CAMPUS

MATA26Y

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TERM TEST II

1. Find the indicated derivatives.

[4] (a) $x = \frac{\sin t}{1 + \cos t}$. Find $\frac{dx}{dt}$.

[4] (b) $y = e^{\cos^2 \theta}$. Find $\frac{dy}{d\theta}$.

[4] (c) $y = u\sqrt{u+1}$, $u = 2x^2 + 3$. Find $\frac{dy}{du}$, and $\frac{dy}{dx}$.

[4] (d) $p(x) = \ln((x-a)(x-b)(x-c))$. a, b, c are constants. Find $p'(x)$.

[4] (e) $(x+y)^2 = (2x+1)^3$. Find $\frac{dy}{dx}$. (Use implicit differentiation. Your answer will involve both x and y .)

2. According to the GLOBE AND MAIL of January 10, 1996, Statistics Canada predicts that Canada's population will be 29 914 300 by July 1, 1996 and 30 269 900 one year later. Assuming that the population growth during that time can be modelled by an exponential function, answer the following.

[2] (a) What is the corresponding rate of growth of Canada's population per year?

[3] (b) What is the expected doubling time?

[5] (c) On which date (plus or minus a day) will Canada's population hit 30 000 000?

[4] 3. (a) State the First Derivative Test for Local Maxima and Minima.

[6] (b) Let f be a function. Is the following sentence true or false?

The inflection points of f are the local extrema of f' .

Explain your answer in a couple of short, clear sentences. You may assume that the second derivative of f is defined and continuous everywhere.

4. Consider $f(x) = x^2e^{-x}$ for $-1 \leq x \leq 3$.

[3] (a) Show that $f'(x) = e^{-x}(2x - x^2)$ and that $f''(x) = e^{-x}(x^2 - 4x + 2)$.

[4] (b) For which x is f increasing? For which is f decreasing?

[4] (c) Find the values of x where $f(x)$ is the greatest; where $f(x)$ is the least.

- [4] (d) Find all values of x where there is a point of inflection.
- [5] (e) Find the values of x where f is increasing most rapidly; where f is decreasing most rapidly.
- [10] (f) Sketch the graph of f , indicating clearly all the data you gathered in (b) through (e).
- [10] 5. Estimate $\int_8^{10} \ln x \, dx$ with accuracy 0.1. Show why you chose the number n of subdivisions that you did.
6. Consider the graph $y = e^x$.
- [5] (a) Find the equation of the tangent line to the graph at (a, e^a) .
- [5] (b) Find the x - and y -intercepts of the line in part (a).
- [10] (c) Show that the highest y -intercept of *any* tangent line is $y = 1$. (You may give a geometric argument as long as you say clearly what properties of the graph you are using.)