MATA26Y April 25 1997

FINAL EXAMINATION

[12] 1. Compute the indicated dervative in each of the following.
Note: Simplification of your answer is not required.

(a) $y(x)$ is the function defined by the equation $e^{xy} = x^3 + y^3$. Find $y'(x)$.

(b) $y(x) = \displaystyle\int_{\sqrt{x}}^{2x} \frac{dt}{\sqrt{t^4 + t^2 + 1}}$. Find $y'(x)$.

(c) $y(x) = x^2 \cos(x)$. Find $y^{(90)}(0)$.

[20] 2. Evaluate each of the following.

(a) $\displaystyle\int xe^{4x}\,dx$

(b) $\displaystyle\int_1^e \frac{(\ln x)^3}{x}\,dx$

(c) $\displaystyle\int \frac{dx}{x^2 - 7x + 6}$

(d) $\displaystyle\int \frac{dx}{e^x + e^{-x}}$

(e) $\displaystyle\int_1^\infty e^{-3x}\,dx$

[4] 3. By making an appropriate trigonometric substitution, rewrite the definite integral $\displaystyle\int_0^3 \sqrt{x^2 + 9}\,dx$ as an integral in terms of the angle θ.
Note: It is not necessary to evaluate the integral.

[6] 4. Let $f(x) = \displaystyle\int_3^x \frac{dt}{\sqrt{t^4 + 3t^2 + 13}}$.

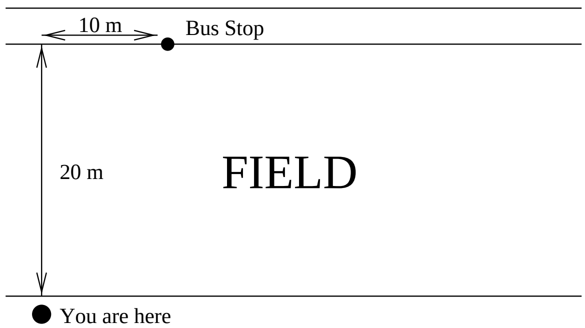
(a) Show that $f(x)$ is invertible.

(b) Compute $g'(0)$ where g is the inverse function to f.

[6] 5. Determine if $\displaystyle\int_1^\infty \frac{4 + \cos(x)}{\sqrt{x^3 + x + 1}}\,dx$ converges or diverges.

[5] 6. Find the radius of convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{nx^{2n}}{2^n}$.

[10] 7. Suppose you wish to reach a bus stop located on the street parallel to the one you are on. The distance between the streets is 20 meters. The region between the streets is a snow-filled field. The bus stop is located 10 meters down the street from the point directly across the field from your present position. Suppose that you can run at 1 meter per second over the snow, but that the street containing the bus stop is plowed so you can run at 3 meters per second along that street. Find the point on the opposite street to aim for to get to the bus stop the fastest.



[10] 8. Let R be the region bounded by the curves $y = x + 1$, $y = 1 - x^3$ and $x = 1$.

(a) Find the area of R.

(b) Find the volume of the figure obtained by revolving R about the x-axis.

[8] 9. Compute $e^{1/2}$ (without calculator except to do addition, subtraction, multiplication, and division) to within 0.05 by using an appropriately chosen Taylor polynomial.

[6] 10. In approximating $\int_a^b f(x)\,dx$, the following values were obtained (in some order) for the left Riemann sum approximation, trapezoidal rule approximation, and midpoint rule approximation, using the same number of subdivisions in each case.

(I) 0.36735

(II) 0.36814

(III) 0.33575

Suppose that f is known to be increasing and concave down.

(a) Determine which approximation is which.

(b) What is the value of the right Riemann sum approximation for this number of subdivisions?

(c) Between which of these approximations does the actual value of the integral lie?

[6] 11. Find the Taylor expansion about 0 for each of the following.

(a) $\sin(x^3)$

(b) $x \arctan x$

[8] 12. (a) Find the solution of $x^2y' - y = 6$ which satisfies $y = 7$ when $x = 1$. Write the solution by giving y as an explicit function of x.

(b) Find the general solution of $y'' + 7y' + 12y = 0$.