Physical Sciences Division
University of Toronto at Scarborough

MATA26Y

October 28, 1996
110 minutes

TERM TEST I

[20] 1. (a) Compute the derivative $f'(x)$ for each of the following functions $f(x)$.
Note: Simplification of your answer is not required.

(i) $f(x) = \dfrac{5x+3}{2x^2+7}$

(ii) $f(x) = x^{1/3}(3x^2+1)$

(iii) $f(x) = \sqrt{x^2+x+3}$

(iv) $f(x) = (\sqrt{x^2+1}+2x)^{25}$

[5] (b) Use the **definition** of derivative to compute the derivative $f'(x)$ for the function $f(x) = 1/x$.
Note: In part (b), NO credit will be given for computations which use the power rule rather than the definition of derivative.

[4] 2. Find the interval(s) on which $f(x) = \dfrac{x^3}{3} - \dfrac{5x^2}{2} + 4x + 1$ is increasing.

[5] 3. (a) Find all values of x which satisfy $3^{(x^2)} = 9^x$.

[8] (b) Suppose a bacterial culture grows exponentially. Assume that initially it contains 2000 organisms and that the amount doubles every 40 minutes. How long will it take until there are 10000 organisms?

[10] 4. Find the equation of the tangent line to $f(x) = x^3 + 2x + 10$ at the point where $x = 2$.

[5] 5. Find $\sin(\tan^{-1}(x+1))$.

[10] 6. Show that $x^5 + x - 1$ has precisely one root r and then determine if $r < 1$ or $r > 1$.

[8] 7. Let $f(x) = x^3 + x + 1$.

(a) Show that $f(x)$ is invertible.

(b) Compute $g'(3)$ where g is the inverse function to f.

[8] 8. Find all values of x which satisfy the following inequalities.

(a) $\dfrac{2x}{x-1} > 1$

(b) $\dfrac{2x}{|x-1|} > 1$

[8] 9. Let $f(x) = |x - 1| - 1$. State the hypotheses and conclusion of Rolle's theorem and explain why this $f(x)$ does not contradict the theorem even though $f(0) = 0$, $f(2) = 0$, and there is no z in $(0, 2)$ for which $f'(z) = 0$.

(You may accept the preceding statement about the derivative of f as fact; you do not have to prove it.)

[10] 10. Let $f(x) = \dfrac{1}{x-1}$.

(a) Find the linear approximation $L(x)$ based at $x = 3$ to $f(x)$.

(b) Find an interval $(3 - h, 3 + h)$ containing 3 throughout which the error in approximating $f(x)$ by its linear approximation based at $x = 3$ is less than 0.001.