

**Physical Sciences Division
University of Toronto at Scarborough**

MATA26Y

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110 minutes

TERM TEST II

- [20] 1. Find dy/dx in each of the following.

Note: Simplification of your answer is not required.

(a) $y = 3^{(x^2)}$

(b) $e^{xy} = x^2 + y^2$

(c) $y = (2 + \sin x)^{(x^4)}$

(d) $y = \int_1^{x^3} \sqrt{1+t^4} dt$

(e) $y = \begin{cases} \frac{e^{3x}-1}{x} & \text{if } x \neq 0; \\ 3 & \text{if } x = 0. \end{cases}$

- [15] 2. Compute each of the following limits.

Note: Exact answers along with the calculations leading to the answer are expected. NO CREDIT will be given for approximating the limit by evaluating the function at nearby points.

(a) $\lim_{x \rightarrow 0} \frac{e^x - \ln(x+e)}{e^x - 1}$

(b) $\lim_{x \rightarrow 1} (2-x)^{\left(\frac{1}{1-x}\right)}$

(c) $\lim_{x \rightarrow 0^+} x^2 \ln x$

- [6] 3. What is the 4th degree Taylor polynomial of e^{2x} based at 1?

Note: Simplification of your answer is not required.

- [10] 4. Find all local extrema of $f(x) = x - 3x^{2/3}$ and determine which are local minimums and which are local maximums.

- [6] 5. The sum of one number and two times a second number is 40. Find the two such numbers with the largest possible product.

- [12] 6. A piece of wire 17 ft long is to be cut into two pieces. One piece will be used to form the boundary of a square, and the other to form the boundary of a rectangle in which one side is twice as long as the other. Find the way to cut the wire which produces a square and rectangle with the smallest possible combined area and find the way which produces the largest possible combined area.

Note: It is permissible to place the “cut” at one end or the other resulting in one of the figures having sides of length 0 if you think that is the optimal solution.

[11] 7. Let $f(x) = e^x + x - 3$.

(a) Show that $f(x)$ has precisely one root.

(b) Find a suitable starting value for application of Newton's method to this function which would guarantee convergence of the iterations to the root from this starting value. You are expected to show your work in obtaining your starting value, which will include the reason that you know that it suitable to guarantee convergence.

Note: It is not required for you to actually perform the iterations on the starting value which find the approximation to the root.

[6] 8. What is the left Riemann sum for $\int_0^{10} f(x)dx$ using a partition into 5 equal subdivisions?

[15] 9. Compute the following:

(a) $\int \sin(3x)dx$

(b) $\int (x + 3)(x + 2)dx$

(c) $\int_2^5 \frac{x^4 + x^2 + 1}{x^3} dx$