

**Physical Sciences Division  
University of Toronto at Scarborough**

MATA26Y

April 23 1998  
3 hours

FINAL EXAMINATION

1. Find all roots of

$$xe^x = 5$$

to at least one decimal place using the method of bisection.

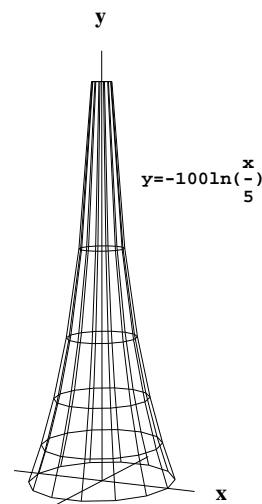
2. Compute the following integrals:

(a)  $\int_{\pi/6}^{\pi/2} \frac{d\theta}{\tan \theta}$       (b)  $\int_{-\pi}^{\pi} e^{2x} \cos 2x \, dx$       (c)  $\int \frac{x^2}{(1+x^2)} dx$

(d)  $\int_0^6 \frac{dx}{(x-4)^{2/3}}$       (e)  $\int_0^{\infty} \frac{x}{e^x} dx$

3. Find the dimensions of the rectangle of largest area that can be inscribed in a semi-circle having diameter 2.

4. In a recent archaeological expedition, a scroll was discovered containing a description of a plan to build what appears to be the Tower of Babel. According to the manuscript, the tower was supposed to have a circular cross section and "go up to the heavens" (i.e., be infinitely high). A mathematician was consulted to solve some of the questions posed by the archaeologists. The mathematicians plotted the silhouette of the lower part of the tower on a set of coordinate axis with the  $y$ -axis running through the center and discovered that it was approximated by the curve  $y = -100 \ln\left(\frac{x}{5}\right)$ . Please answer the questions posed by the archaeologists:



Would such a tower have finite volume? If so, what would the volume be?

5. (a) Show that

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$$

converges.

- (b) How many terms are needed to compute the sum in part (a) to within  $10^{-4}$ ? (Do not compute the partial sum.)
- (c) Do the following series converge or diverge?

$$\sum_{n=1}^{\infty} \frac{1}{1+3n^2} \qquad \sum_{n=1}^{\infty} \frac{1}{n^2} \left( \sin \frac{\pi}{n} \right) \qquad \sum_{n=1}^{\infty} \frac{(2n)!}{n!n!} .$$

6. (a) Find the Taylor series of  $f(t) = te^t$  at  $t = 0$ .
- (b) Using your answer to (a), find the Taylor series of

$$F(x) = \int_0^x te^t dt$$

at  $x = 0$ .

- (c) From your solution to (b) show that

$$\sum_{n=0}^{\infty} \frac{1}{(n+2)(n!)} = 1 .$$