

**Physical Sciences Division  
University of Toronto at Scarborough**

MATA26Y

November 5, 1997  
110 minutes

TERM TEST I

1. A wire of length  $L$  is cut into two pieces (we allow the possibility that one of the pieces has zero length). The first piece is bent into a circle, the second into a square. How long should the piece of wire that is bent into a circular shape be in order to maximize the *sum* of the areas of the two shapes? How long should that piece be in order to minimize the sum of the two areas?
2. Find  $f'(x)$  for the following functions:
  - (a)  $f(x) = e^{\sin(x^5+3x^3+2x+1)}$
  - (b)  $f(x) = x^\pi$
  - (c)  $f(x) = \sin(\cos \sqrt{1-x^2}) \quad x \in [-1, 1]$
  - (d)  $f(x) = \frac{x^{1/3}(\ln x)}{x^2 + 3}$
  - (e)  $f(x) = \arccos x$
3.
  - (a) Let  $f$  be an even function such that  $f(0) \neq 0$ . Is it possible for  $f$  to have exactly 121 roots? Justify your answer.
  - (b) Let  $g$  be an odd function. What is the minimum number of roots of  $g$ ? Justify your answer.
4. Let  $f(x) = 2x - 3 + \ln x$ .

How many roots does  $f$  have? (Justify your answer.)

For each of the roots of  $f$  find an interval  $[u, v]$  and constants  $M_1$ ,  $M_2$  and  $K$  for which Newton's method is guaranteed to work. In each case find the number  $n$  of iterations necessary to find the root with an error of at most  $10^{-5}$ .
5.
  - (a) How many real roots does  $f(x) = x^4 + 4x - 1$  have?
  - (b) Compute each root to within 0.1. (Do NOT use Newton's method.)
  - (c) Give the statement of any theorem that you use.
  - (d) How many real roots does  $x^4 + ax + b$  have,  $a, b \in \mathbb{R}$ ?
6. (a) Assuming  $x \geq 0$  and  $a > 0$ , find the maximum and minimum values of

$$y = e^{-ax} \sin x$$

in terms of  $a$ .

- (b) Describe in words how the position and values of these maxima and minima change if  $a$  increases. Give an intuitive justification for this.