Physical Sciences Division
University of Toronto at Scarborough

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TERM TEST II

1. Let
$$I = \int_0^{\pi/2} \cos(x^2)dx \ .$$

   (a) Find $n > 0$ so that
$$|\,\mathrm{Simp}(n) - I| \leq 10^{-5} \ .$$
   The error bound for Simpsons Rule for $\int_a^b f(x)dx$ is
$$\frac{M(b-a)^5}{16 \cdot 180 n^4}$$
   where $M \geq |f^{(4)}(x)|$ for all $x \in [a,b]$. Compute $\mathrm{Simp}(n)$.

   (b) How large would n have to be to achieve the same accuracy with the Trapezoidal Rule?
   $\left(\text{error bound is } \frac{M(b-a)^3}{12n^2}\ , \quad M \geq |f''(x)|,\ x \in [a,b]\right)$.

   (c) How large would n have to be to achieve the same accuracy with the Right Hand Rule?
   $\left(\text{error bound is } \frac{M(b-a)^2}{2n}\ , \quad M \geq |f'(x)|,\ x \in [a,b]\right)$.

2. Use a Taylor polynomial to approximate $\ln 1.1$ to within 10^{-5}.

3. Compute the following integrals:

   (a) $\displaystyle\int (\ln x)^2 dx$

   (b) $\displaystyle\int \frac{4+x}{\sqrt{x}}\, dx$

   (c) $\displaystyle\int \frac{\ln(1+y)}{1+y}\, dy$

   (d) $\displaystyle\int \frac{x\, dx}{x^2+3x+2}$

4. (a) Find the average value of the following functions:

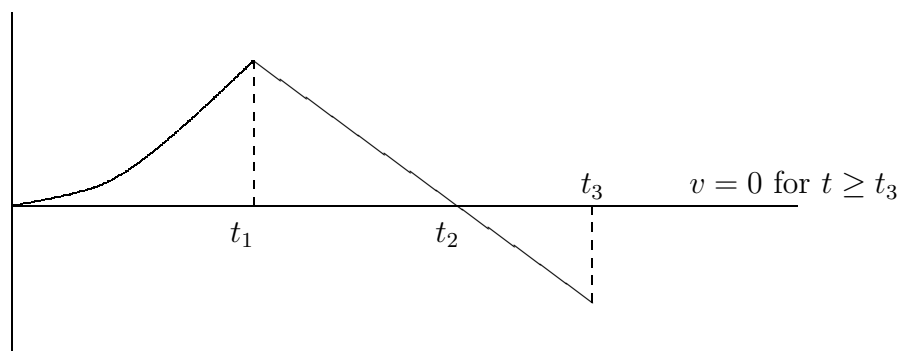
   (i) $f(t) = \cos t \qquad t \in [0, 2\pi]$

   (ii) $g(t) = |\cos t| \qquad t \in [0, 2\pi]$

   (iii) $k(t) = (\cos t)^2 \qquad t \in [0, 2\pi]$

   (b) Write the averages you have just found in ascending order. Explain clearly, using words and graphs, why the averages should be expected to come out in the order they did.

5. A young girl who aspires to be a rocket scientist launches a model rocket from the ground at time $t = 0$. The rocket travels straight up in the air, and the following graph shows the upward velocity of the rocket as a function of time:



Let h be the height (or vertical displacement) of the rocket, let a be the acceleration of the rocket, and let $h = 0$ be the ground level from which the rocket was launched. Recall that the first derivative of displacement is velocity and that the first derivative of velocity is acceleration.

   (a) Sketch a graph of the acceleration of the rocket as a function of time.

   (b) Sketch a graph of the height of the rocket as a function of time.

   (c) Let $v(t)$ be the function that gives the velocity of the rocket at time t. From the graph of the rocket's velocity, which is larger, $\left|\int_0^{t_2} v(t)dt\right|$ or $\left|\int_{t_2}^{t_3} v(t)dt\right|$? What do you know about the sign of $\int_0^{t_3} v(t)dt$? What does this mean physically?