

**Physical Sciences Division
University of Toronto at Scarborough**

MATA26Y

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110 minutes

TERM TEST I

- [14] 1. Using only the definition of derivative, find $f'(x)$ for

$$f(x) = \frac{1}{x-1}.$$

- [15] 2. Evaluate the following limits:

(a) $\lim_{x \rightarrow 0^-} \frac{x}{|x|}$

(b) $\lim_{x \rightarrow 0} \frac{\sin 2x}{\cos 3x}$

(c) $\lim_{x \rightarrow 0} \frac{e^x - x - 1}{x^2}$

- [20] 3. (a) State the Mean Value Theorem
(b) Give an example of a function which is continuous on $[-2, 0]$ and to which the Mean Value Theorem does not apply.
(c) Define formally what is meant by each of the following:
(i) $\lim_{x \rightarrow a} f(x) = L$
(ii) The function $f(x)$ is continuous at $x = a$.

- [15] 4. Determine $\frac{dy}{dx}$ in each of the following cases:

(a) $y = \frac{x^3 + x - 3}{\sin x + \ln x}$

(b) $y = \ln(\sin(x^3 + x - 3))$

(c) $\ln y + \sin y = x^3 + x - 3.$

- [15] 5. Determine all intervals on which

$$f(x) = 3x^{\frac{2}{5}} + x^{\frac{7}{5}}$$

is increasing and decreasing.

- [15] 6. Write the equation of the tangent line to $f(x) = x^{\sqrt{x}}$ at $x = 4$.

[6] The A⁺ Question

Determine $\lim_{x \rightarrow \infty} (x - \ln x)$, if the limit exists. Justify your conclusion.