

**Physical Sciences Division**  
**University of Toronto at Scarborough**

MATA26Y

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110 minutes

TERM TEST II

- [16] 1. Sketch the graph of  $f(x) = x^2 e^{-x}$ .
- [12] 2. Two towns on opposite sides of a river want to be joined by a telephone line. The river is perfectly straight and 1 km wide. One town is exactly 3 km upstream from the other. Suppose laying underwater telephone cable costs \$100,000 per km but laying cable on dry land costs only \$50,000 per km. What will be the least expensive route for the cable?
- [35] 3. Evaluate the following integrals:
- |   |   |
|---|---|
| (a) $\int \sqrt{x} \ln x \, dx$           | (d) $\int_0^{\frac{\pi}{4}} \frac{\sin x}{\sqrt{\cos x}} \, dx$ |
| (b) $\int \frac{x^2}{\sqrt{1-x^2}} \, dx$ | (e) $\int_{-3}^1  x^3 - x  \, dx$                               |
| (c) $\int \frac{x+1}{x^4+x^2} \, dx$      |   |
- [12] 4. Describe how many subdivisions  $n$  (into equal subintervals) you would use to approximate within  $10^{-5}$  the integral
- $$\int_1^2 \frac{dx}{3+2x}$$
- using (a) the trapezoid rule and (b) Simpson's rule.  
(Do not work out the approximations).
- [15] 5. Approximate a solution of  $x + \sin x = 1$  within  $10^{-4}$  using Newton's method.

The A<sup>+</sup> Question

- [10] Use mathematical induction to prove that
- $$\sum_{i=1}^n i^3 = \left[ \frac{n(n+1)}{2} \right]^2 .$$