University of Toronto at Scarborough
Physical Sciences Division, Mathematics

MATA26Y

November 10, 1999
110 minutes

TERM TEST I

1. Let P be a polynomial.
   (a) [3 *pts*] State the Factor Theorem which gives a necessary and sufficient condition for $(x-a)$ to be a factor of $P(x)$.
   (b) [2 *pts*] Use the Factor Theorem to show that $x+1$ divides $x^{1001}+1$. (Do not attempt to calculate the quotient.)
2. Let $f = \frac{P}{Q}$ be a rational function (where P and Q are polynomials).
   (a) [1 *pts*] Define "domain of f" .
   (b) [1 *pts*] Define "singularities of f" .
   (c) [2 *pts*] State a theorem giving information about the number of roots and singularities of f.
   (d) [4 *pts*] Find the *domain*, the *order* and *all the roots and singularities* of the function
   $$f(x) = 1 + \frac{1}{x-1} - \frac{1}{x^2+1} .$$
3. Let f be a rational function.
   (a) [5 *pts*] State the Intermediate Value Theorem (IVT) for f on the interval $[a,b]$.
   (b) [5 *pts*] What does the IVT have to say about the function
   $$f(x) = \frac{x^3+1}{(x^2+1)(x-1)}$$
   for the interval $[0,2]$?
4. Let f be a rational function.
   (a) [5 *pts*] State the Rolle's theorem for f on the interval $[a,b]$.
   (b) [5 *pts*] What does Rolle's theorem have to say about the function
   $$f(x) = \frac{x(x+1)}{x^2+1}$$
   on the interval $[-1,0]$? Verify this by actual calculation.
5. Let f be a rational function.
   (a) [5 *pts*] State the Mean Value Theorem (MVT) for f on the interval $[a,b]$.
   (b) [5 *pts*] What does MVT have to say about the function
   $$f(x) = \frac{x^3+2x-3}{x+2}$$
   on the interval $[0,1]$?

6. Let $P(x) = x^6 + 5x^3 - x - 1$.
   (a) [3 *pts*] Find the linear approximation $A_1(x)$ to P near the point $a = 1$.
   (b) [5 *pts*] Find $h > 0$ such that $|P(x) - A_1(x)| \leq .001$ for $|x - 1| \leq h$.

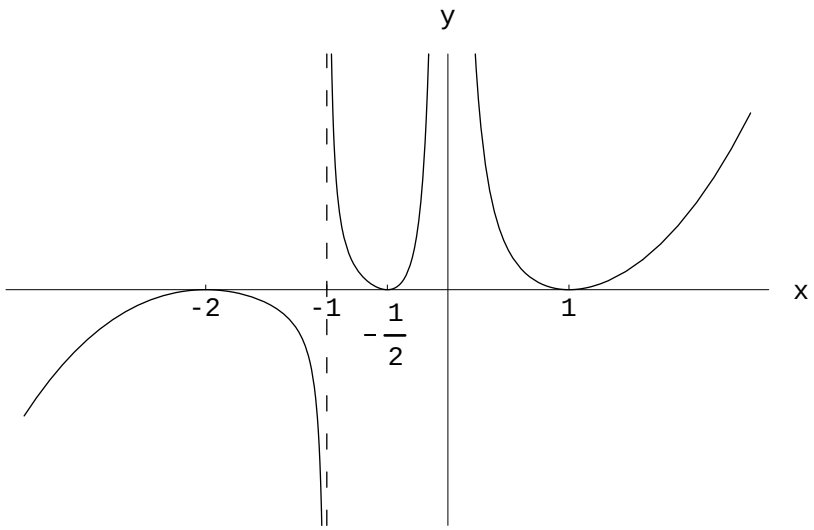
7. [15 *pts*] Use Newton's method to find all the roots of
$$f(x) = x^3 - 6x^2 + 15x - 2$$
to an accuracy of 10^{-5}.

8. Let
$$f(x) = \frac{x^4 + 2}{x^2 - 1}$$
   (a) [2 *pts*] State the order of f.
   (b) [2 *pts*] Evaluate $\lim_{x\to\infty} f(x)$.
   (c) [4 *pts*] Find positive constants R, m, M such that
$$m|x|^r \leq |f(x)| \leq M|x|^r$$
   for $|x| \geq R$, where $r = \text{order} f$.

9. [8 *pts*] If $f(x)$ and $g(x)$ are rational functions with order $f = r$ and order $g = s$, prove that order $(f/g) = r - s$.

10. [10 *pts*] Use the chart method for inequalities to solve
$$\frac{|x^2 - 1|(x^2 - x - 6)}{x^2 - 25} \leq 0$$

11. [8 *pts*] Find a formula for a rational function $f(x)$ having a graph of the form



(Assume that $f(x) \to \infty$ as $x \to \infty$, and $f(x) \to -\infty$ as $x \to -\infty$.)