University of Toronto at Scarborough
Physical Sciences Division, Mathematics

MATA26Y

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110 minutes

TERM TEST II

PART A (Multiple Choice)

Circle the correct answer. *(Circling more than one answer will void the question.)* In the case of Question 5, calculate the root. Each question is worth 6 marks.

1. If $f(x) = \sin(\tan(\cos(x^2)))$ then $f'(x)$ is:

   (a) $\cos(\tan(\cos(x^2)))$, (b) $2x$, (c) $\cos(\sec^2(-\sin(2x)))$, (d) none of these

2. The domain of $f(x) = \ln(2x^4 + 1)$ is:

   (a) $(0, \infty)$, (b) $[0, \infty)$, (c) $\mathbb{R}$, (d) none of these

3. If $y^3 + x^3 + 3xy = 15$ then $\dfrac{dy}{dx}$ at $(x_0, y_0) = (1, 2)$ is

   (a) $3/5$, (b) $-3/5$, (c) $5/3$, (d) $-5/3$, (e) none of these

4. $\lim_{x\to 0}(1 + \sin x)^{1/x}$ is

   (a) e, (b) 1, (c) doesn't exist, (d) none of these

5. It is known that the equation $x = 6\sin x$ has just one positive root. Evaluate this root to within an error of $\pm 5 \times 10^{-5}$.

   root = $\qquad\qquad \pm 5 \times 10^{-5}$

6. $\lim_{x\to 0} \dfrac{(\tan x) - x - \frac{1}{3}x^3}{x^5}$ is

   (a) $\dfrac{1}{15}$, (b) $\dfrac{2}{15}$, (c) 0, (d) doesn't exist (e) none of these

7. Which of the following is the Riemann sum for $\int_1^4 x^2\,dx$ arising from a partition of [1,4] into equal subintervals with the midpoint selection?

(a) $\sum_{k=1}^{n}\left(1+\frac{k}{n}\right)^2\frac{1}{n}$, (b) $\sum_{k=1}^{n}\left(\frac{2(k+n)-1}{2n}\right)^2\frac{1}{n}$, (c) $\sum_{k=1}^{n}\left(\frac{2(k+n)+1}{2n}\right)^2\frac{1}{n}$,

(d) 21, (e) none of these

8. Using the formula $1^2+2^2+\cdots+n^2=\frac{1}{6}n(n+1)(2n+1)$, the expression (a) in question 7 simplifies to

(a) $\frac{1}{3}$, (b) $\frac{1}{3}+\frac{1}{2n}+\frac{1}{6n^2}$, (c) 21, (d) none of these

9. The area under the graph of $f(x)=1-x^2$ on the interval $[-1,1]$ is

(a) $\frac{4}{3}$, (b) $\frac{3}{4}$, (c) 2, (d) doesn't exist, (e) none of these

10. Let $F(t)=\int_0^{1/t}\frac{dx}{1-x^3}$. Then $F'(2)$ is

(a) $-\frac{1}{7}$, (b) $\frac{8}{7}$, (c) 0, (d) $-\frac{2}{7}$, (e) none of these

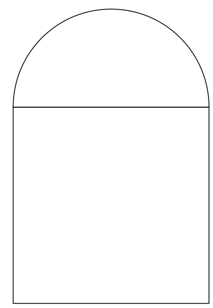
Part B

[10] 11. Find constants a, b, c, d so that the function

$$f(x) = \begin{cases} \dfrac{\sin ax}{bx} & , \quad x < 0 \\ ax + 1 & , \quad 0 \le x < 1 \\ cx^2 - 2 & , \quad 1 \le x < 2 \\ \dfrac{d(x^2-4)}{\sqrt{x}} & , \quad 2 \le x < 4 \\ 12 & , \quad x \ge 4 \end{cases}$$

is continuous.

[10] 12. A Norman window has the shape of a rectangle surmounted by a semicircle. Find the maximum area of a Norman window with 12 foot perimeter.



[10] 13. Let $f(x) = (x^2 - 1)e^x$.

(a) On what interval is $f(x)$ negative?

(b) On what interval is $f(x)$ decreasing?

(c) On what interval is $f(x)$ concave downward?

(d) Sketch the graph of $f(x)$.

[8] 14. State the Fundamental Theorem of Calculus.