University of Toronto at Scarborough
Physical Sciences Division, Mathematics

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110 minutes

Term Test #1

1. Evaluate the limits:

[**4**] (a) $\lim_{x \to 0} \dfrac{x}{2(1-\sqrt{x+1})}$

[**4**] (b) $\lim_{x \to 0} \dfrac{1-\cos 2x}{x \sin 3x}$

2. Simplify the functions:

[**4**] (a) $\sin(\arctan x)$

[**4**] (b) $1-\sin^2\Big(\arccos\big((x+1)^2\big)\Big)$

3. Let $f(x) = \dfrac{\sqrt{5x^2-2}}{x+3}$.

[**2**] (a) Find $\lim_{x \to \infty} f(x)$

[**2**] (b) Find $\lim_{x \to -\infty} f(x)$

4. Let $f(x) = \dfrac{(x-4)(x^2-4x-5)}{(x^2-2x-3)(4-x^2)}$. Find

[**2**] (a) the domain of $f(x)$.

[**2**] (b) the roots of $f(x)$.

[**4**] (c) all x such that $f(x) > 0$.

[**4**] (d) all x such that $f(x) < 0$.

[**8**] 5. Find a nonzero value for the constant k so that the function $f(x)$ defined by

$$f(x) = \begin{cases} \dfrac{\tan(kx)}{x} & x < 0 \\ 3x + 2k^2 & x \geq 0 \end{cases}$$

will be continuous at $x = 0$.

6. Find the indicated derivatives.

[4] (a) $f(x) = \sqrt{\dfrac{x}{2} + \sqrt{\dfrac{2}{x}}}$; $f'(x)$

[6] (b) $f(x) = \sin^2(\arctan(\tan x^2))$; $f''(x)$
(*Hint:* simplify $f(x)$ first)

[6] (c) $(x + y) = \tan(x + 4y^2)$; $\left.\dfrac{dy}{dx}\right|_{(0,0)}$

[4] 7. (a) State Rolle's Theorem.

[4] (b) Find exactly how many positive roots the function $f(x) = \dfrac{1}{(x+1)^3} - 3x + \sin x$ has. *(Do not calculate the root.)*

[8] 8. For the function $f(x) = \dfrac{2x}{x^4 - 1}$ find positive numbers R, m, M such that

$$|x|^r m \le |f(x)| \le |x|^r M$$

for all x satisfying $|x| \ge R$. ($r = \text{order}(f)$)

9. Let $f(x) = (1 + x)\sin x$.

[2] (a) Find the constant approximation $f_0(x)$ to $f(x)$ at $x = \frac{1}{2}\pi$.

[6] (b) Find $h > 0$ such that $\left|x - \frac{1}{2}\pi\right| \le h \Rightarrow \left|f(x) - f_0(x)\right| \le \frac{1}{10}$.

[2] (c) Find the linear approximation $f_1(x)$ to $f(x)$ at $x = \frac{1}{2}\pi$.

[6] (d) Find $h > 0$ such that $\left|x - \frac{1}{2}\pi\right| \le h \Rightarrow \left|f(x) - f_1(x)\right| \le \frac{1}{10}$.

10. Consider the equation $\tan x = x$ where x is in radians.

[4] (a) Show it has exactly one root r on the interval $[4.4, 4.6]$.

[4] (b) Use Newton's method, starting with $r_0 = 4.5$ to calculate r_1, r_2, r_3 to five decimal places.

[4] (c) Show that if $\bar{r}_3$ is r_3 rounded to five decimal places, then $|r - \bar{r}_3| \le .000005$.