UNIVERSITY OF TORONTO AT SCARBOROUGH
PHYSICAL SCIENCES DIVISION, Mathematics

MATA26Y February 7, 2001

Term Test II

[14] 1. Sketch the graph of the function $f(x) = x^4 - 2x^2$, showing extrema, points of inflection, intervals of increase and decrease and intervals of concavity.

2. Calculate the following.

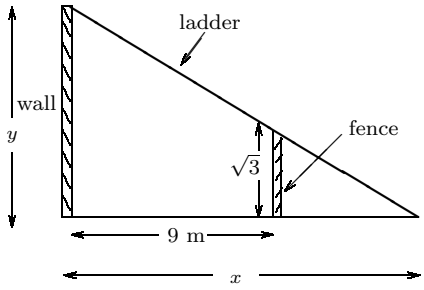
[4] (a) $\lim_{x\to\infty} x\left(2^{\frac{1}{x}} - 1\right)$

[4] (b) $\lim_{x\to 1} \dfrac{\sqrt{x^2+x+1} - 3}{x}$

[6] (c) $\lim_{x\to 0} \dfrac{x\cos x - \sin x}{x\sin^2 x}$

[6] (d) $\dfrac{d}{dx}\left((\ln x)^{\arctan x}\right)$

[12] 3. Find the length of the shortest ladder that can extend from a vertical wall, over a vertical fence $\sqrt{3}$ m high located 9 m away from the wall, to a point on the ground outside the fence.



[4] 4. (a) State the Fundamental Theorem of Calculus.

[8] (b) Find the area bounded by $f(x) = 4 - x^2$ and the x–axis on $[-4, 4]$.

[4] (c) Find $f'(x)$ where

$$f(x) = \int_{\sin x}^{\sqrt{1+x^2}} e^{t^2} dt \ .$$

[8] 5. A function f satisfies $|f^{(4)}(x)| \leq 7$ on the interval $[1, 3]$, and has values $f(1.0) = 0.1860$, $f(1.5) = 0.9411$, $f(2.0) = 1.1550$, $f(2.5) = 1.4511$, and $f(3.0) = 1.2144$. Find the best possible Simpson's rule approximation to $I = \int_1^3 f(x)dx$, based on this data. Give a bound for the size of the error, and specify the smallest interval you can that must contain the value of I.

6. Calculate each of the following integrals.

[4] (a) $I = \int_0^{\pi} \cos\left(\frac{1}{2}\theta\right) d\theta$

[4] (b) $I = \int x e^x dx$

[4] (c) $I = \int \sqrt{1-2x}\, dx$

[4] (d) $I = \int \dfrac{dx}{x^2-4}$

7. Calculate each of the following integrals.

[4] (a) $I = \int_0^{\frac{\sqrt{3}}{2}} x\sqrt{1-x^2}\, dx$

[6] (b) $I = \int_0^{\frac{\sqrt{3}}{2}} \sqrt{1-x^2}\, dx$

[4] (c) $I = \int e^x \sin x\, dx$