

**University of Toronto at Scarborough
Physical Sciences Division, Mathematics**

**First Midterm Test
MATA26Y
Calculus**

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Duration: 110 minutes

1. Evaluate the limit:

(a) [5 points] $\lim_{x \rightarrow 1} \frac{2x^2 + 2x - 4}{x - 1}$

(b) [5 points] $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{12x^2}$

(c) [5 points] $\lim_{x \rightarrow \infty} \frac{\sin x}{x}$

2. Let $f(x) = \frac{x^2 + 2x}{\sqrt{x^3 + 4x^2}}$

(a) [3 points] Find $\lim_{x \rightarrow 0^+} f(x)$

(b) [2 points] Find $\lim_{x \rightarrow 0^-} f(x)$

3. [10 points] For $f(x) = \frac{2x}{3x^2 - 1}$, find positive number R, m, M such that

$$m|x|^r \leq |f(x)| \leq M|x|^r$$

for all x satisfying $|x| \geq R$ where $r = \text{ord } f$.

4. [10 points] Find all possible values of a and b so that $f(x)$ is continuous for all $x \in \mathbb{R}$ if

$$f(x) = \begin{cases} |ax + 3| & \text{if } x \leq -1 \\ |3x + a| & \text{if } -1 < x \leq 0 \\ \frac{b \sin 2x}{x} - 2b & \text{if } 0 < x < \pi \\ \cos^2 x - 3 & \text{if } x \geq \pi \end{cases}$$

5. (a) [4 points] Give the statement of the Mean Value Theorem.
- (b) [6 points] If $h'(x) = \frac{1}{3 + 2x^2}$ for all $x \in \mathbb{R}$ and $h(1) = 0$, show $\frac{1}{11} < h(2) < \frac{1}{5}$.
6. Find the indicated derivatives
- (a) [5 points] $f(x) = x^x$; $f'(x)$
- (b) [5 points] $x^5 - xy + y^3 = 8$; $y'(0)$
- (c) [5 points] $f(x) = (1 + x)^{2/3}(2 - x)^{1/3}(1 + x^2)^{3/2}(1 + \ln x)^{1/2}$; $f'(1)$
- (d) [5 points] $h(x) = \tan(\arccos x)$; $h'(x)$
7. Let $f(x) = x^2 + 2e^{2(x-1)}$.
- (a) [5 points] Find the linear approximation $A_1(x)$ for $f(x)$ at $x = 1$.
- (b) [10 points] Find $h > 0$ such that $|f(x) - A_1(x)| < .01$ for $|x - 1| < h$.
8. Let $P(x) = 10x^4 - 5x - 4$
- (a) [5 points] How many roots does $P(x)$ have?
- (b) [10 points] Determine the smallest root of $P(x)$ to four decimal place accuracy (i.e., to within an error of ± 0.00005)