

**University of Toronto at Scarborough
Physical Sciences Division, Mathematics**

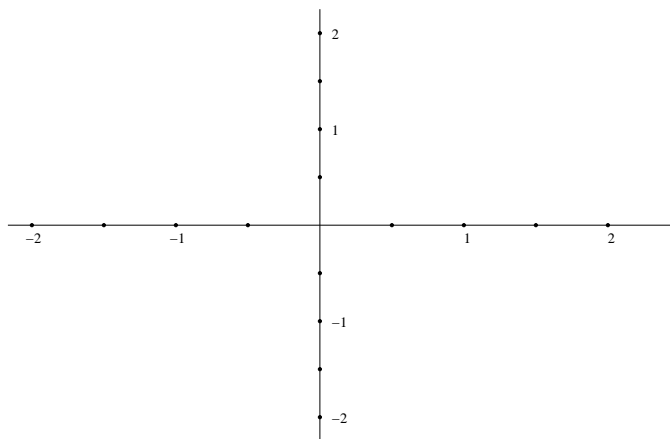
**Second Midterm Test
MATA26Y
Calculus**

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Duration: 110 minutes

1. [24 pts] Let $f(x) = \frac{1}{3}x^3 + \frac{1}{4}x^2 - \frac{45}{64}x$.

- (a) [2] State the domain of f
- (b) [4] Find the intervals where $f(x)$ is positive and those where it is negative.
[Hint: $f(x)$ does factor completely.]
- (c) [4] Determine the intervals of increase and decrease of f .
- (d) [4] Determine the concavity and inflection points of f .
- (e) [4] Determine the local and global extrema of f .
- (f) [6] Sketch the graph of f using the information above. Label your diagram with this information.



2. [12 pts] Calculate the following limits.

$$(a) [4] \lim_{x \rightarrow 0^+} (1+x)^{1/x} \quad (b) [4] \lim_{x \rightarrow 0} \frac{x - \sin x}{\sin^3 x} \quad (c) [4] \lim_{x \rightarrow 3} \frac{x-3}{x+3}$$

3. [12 pts] Ship A is sailing east at 20 k./hr. Ship B is sailing northward at 10 k./hr. At noon, ship B is 250 k. directly south of ship A. At what time are the ships closest?

4. [18 pts]

(a) [4] Write down the Riemann sum for $f(x) = x^2$ over the interval $[0,1]$ for $n = 4$, using the “left-most” selection x_k^* , and evaluate this Riemann sum.

(b) [6] State the Fundamental Theorem of Calculus (both parts).

(c) [4] Find the area under the curve of $y = \sqrt{x} - x^2$ over the interval $[0, 1]$.

(d) [4] Let $F(x) = \int_{-2x}^x e^{-t^2} dt$. Determine $F'(1)$. [Hint: You cannot do this directly by finding $\int e^{-t^2} dt$. Use the Fundamental Theorem of Calculus.]

5. [12 pts] Calculate the following integrals

$$(a) [4] \int \frac{dx}{x+1} \quad (b) [4] \int \frac{dx}{\sqrt{x+1}} \quad (c) [4] \int \frac{dx}{x^2+1}$$

6. [12 pts] Calculate the following integrals

$$(a) [4] \int \frac{x dx}{\sqrt{x^2-1}} \quad (b) [8] \int \frac{4x^2 dx}{(x-2)(x^2+4)}$$

7. [10 pts] State why the following integrals are improper and calculate them.

$$(a) [5] \int_0^{\infty} x e^{-x} dx \quad (b) [5] \int_0^1 \frac{dx}{(1-x)^{2/3}}$$

8. [15 pts] BONUS QUESTION

(a) [3] Verify that $\tan \frac{\pi}{8} = \sqrt{2} - 1$. (Don't use your calculator for this. Use the fact that $\tan \frac{\pi}{4} = 1$ and the half angle formulas.)

(b) [3] Explain why $I = \int_0^{\pi/4} \frac{d\theta}{\sin \theta + \cos \theta - \sqrt{2}}$ is an improper integral.

(c) [9] Calculate I .