University of Toronto at Scarborough
Division of Mathematical Sciences

First Midterm Test
MATA26Y
CALCULUS

Examiners: P. Hill
X. Jiang
S. Tryphonas

Date: November 13, 2002
Duration: 110 minutes

1. **[16 points]** Evaluate the following limits if they exist.
Do not use L'Hôpital's rule.

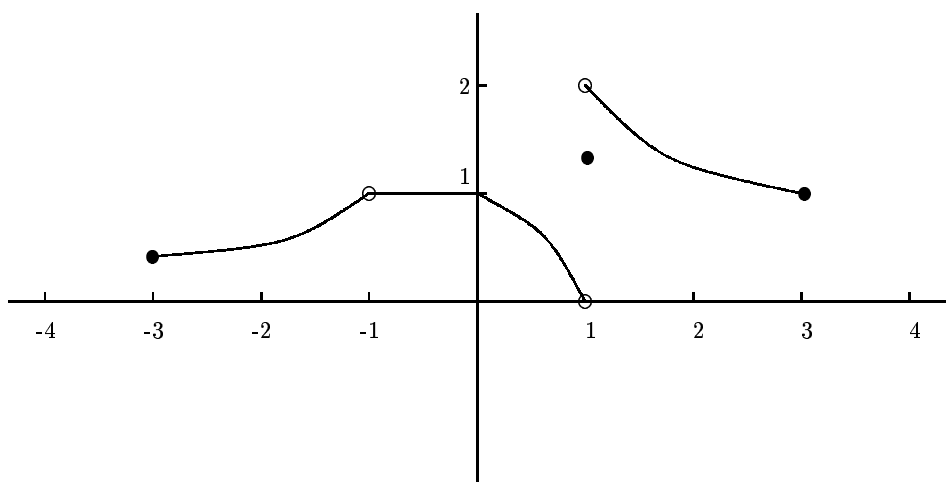
(a) $\displaystyle\lim_{x\to-\infty} \frac{x^3+2x^2+3x+4}{4x^3+3x^2+2x+1}$

(b) $\displaystyle\lim_{\theta\to 0} \frac{\cos\theta - 1}{\theta\sin\theta}$

(c) $\displaystyle\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1}$

(d) $\displaystyle\lim_{x\to 0} \sqrt[3]{x}\cos\frac{1}{x^2}$

2. **[8 points]** The graph of the function f is given below



(a) Evaluate $\displaystyle\lim_{x\to -1^+} f(x)$ and $\displaystyle\lim_{x\to -1^-} f(x)$.

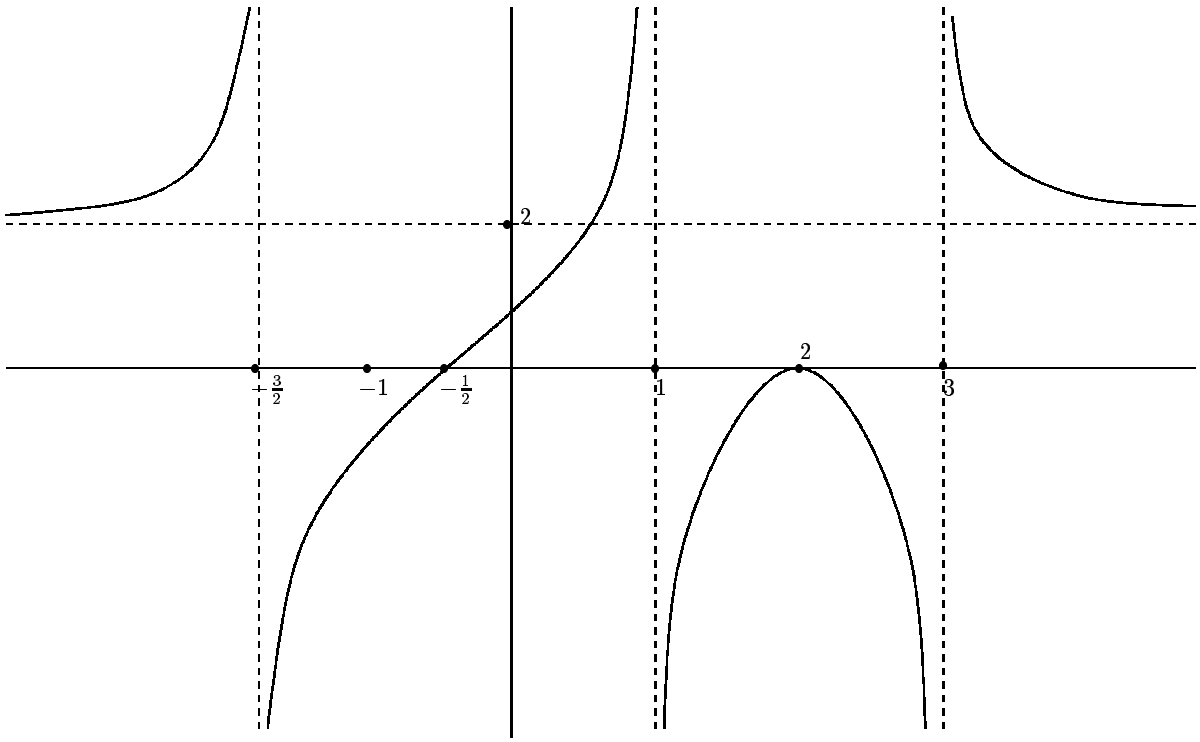
(b) Evaluate $\displaystyle\lim_{x\to 0^+} f(x)$ and $\displaystyle\lim_{x\to 0^-} f(x)$.

(c) Evaluate $\displaystyle\lim_{x\to 1^+} f(x)$ and $\displaystyle\lim_{x\to 1^-} f(x)$.

(d) For what values of x between -3 and 3 is f discontinuous? Find any removable discontinuities of f (if there are any).

3. **[8 points]** Let $f(x) = \dfrac{2x+1}{x^2-1}$ be a rational function. Find constants R, m and M such that $m|x|^r \leq |f(x)| \leq M|x|^r$ for $|x| > R$ where $r = \operatorname{ord} f$.

4. **[8 points]** Given that the graph below is a graph of a rational function. Give a possible formula for that function.



5. **[8 points]** If $g(x) = \dfrac{1}{1+x^2}$, find $g'(x)$ using the definition of the derivative of g at x.

6. **[20 points]** Find the indicated derivatives.

   (a) $f(x) = x(x-2)^{1/3}$ find $f'(x)$.

   (b) $\sin(xy) = \cos(y^2)$ find $\left.\dfrac{dy}{dx}\right|_{(\sqrt{\frac{\pi}{4}}, \sqrt{\frac{\pi}{4}})}$.

   (c) $g(x) = \arcsin(\sin\sqrt{x^2+a})$ find $g'(x)$.

   (d) $h(x) = \dfrac{\tan\sqrt{1+x}}{\sqrt{1+x}}$ find $h'(x)$.

7. **[8 points]**

   (a) State the Mean Value Theorem.

   (b) On what subset of $[0, \pi]$ is $f(x)$ strictly decreasing, where

$$f(x) = \sin x - \left(x - \frac{\pi}{2}\right)\cos x$$

8. **[10 points]** Let $g(x) = x^5 + x^3 + 1$ and let $A_1(x)$ be the linear approximation to g at $a = 1$. Show that $|E_1(x)| < 0.2$ for all $x \in (.9, 1.1)$.

9. **[14 points]** $f(x) = x + x^5 - \cos x$.

   (a) Show f has at most one root.

   (b) Show f has a root in the interval $(0.5, 1)$.

   (c) Find the root of f to within .0005.