University of Toronto at Scarborough
Division of Mathematical Sciences

Second Midterm Test
MATA26Y
CALCULUS

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Date: February 5, 2003
Duration: 110 minutes

1. **[20 points]** Evaluate the following limits if they exist.

   (a) $\lim\limits_{x\to 0} \dfrac{6\sin x - 6x + x^3}{x^5}$

   (b) $\lim\limits_{x\to \infty} (\ln x)^{1/x}$

   (c) $\lim\limits_{x\to 0} \dfrac{\sqrt[3]{1+x} + \sqrt[3]{1-x}}{x}$

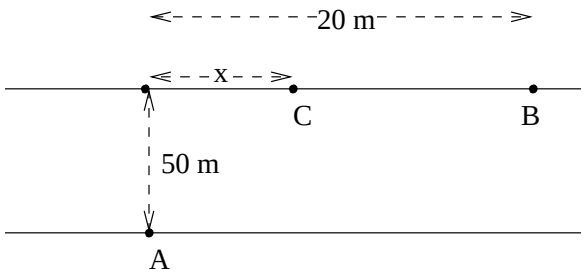
   (d) $\lim\limits_{x\to 1} \dfrac{1 - x + \ln x}{1 + \cos(\pi x)}$

2. **[10 points]** $f(x) = \arctan(\ln x)$ for $x > 0$.

   (a) Show that f is invertible.

   (b) If g is the inverse function of f, find $g'(0)$.

3. **[10 points]** On a nice hot summer day, Jennifer is standing at point A on the side of Rouge Valley river which is 50 m wide. She wants to go to point B which is on the opposite side of the river exactly 20 m upstream. Suppose Jennifer can run twice as fast as she can swim. What route should she take to get to B the fastest?



4. **[12 points]** Graph $f(x) = x^{1/3}(x-1)$ showing:

   i) intervals where f is increasing and
   intervals where f is decreasing.

   ii) intervals where f is concave upwards and
   intervals where f is concave downwards.

   iii) local maxima, local minima, global maxima, global minima and inflection points if they exist.

   iv) give the domain and range of f.

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5. **[10 points]**

   (a) Show that $\dfrac{\left(\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \sqrt{\frac{3}{n}} + \cdots + \sqrt{\frac{n}{n}}\right)}{n}$ is a Riemann sum for $\displaystyle\int_0^1 \sqrt{x}\,dx$

   (b) Conclude that $\displaystyle\lim_{n\to\infty}\left(\frac{1+\sqrt{2}+\sqrt{3}+\cdots+\sqrt{n}}{n^{3/2}}\right) = \frac{2}{3}$.

6. **[10 points]**

   (a) State the Fundamental Theorem of Calculus (both parts).

   (b) Let $F(x) = \displaystyle\int_{\sin x}^{\cos x} e^{(1+\arcsin t)^2}\,dt$. Find $F'(x)$.

7. **[28 points]** Evaluate

   (a) $\displaystyle\int x^2 e^x dx$

   (b) $\displaystyle\int \frac{x^2}{\sqrt{25-x^2}}\,dx$

   (c) $\displaystyle\int_1^4 \frac{(1+\sqrt{x})^4}{\sqrt{x}}\,dx$

   (d) $\displaystyle\int \frac{x^2+2x+2}{(2x+1)(x^2+1)}\,dx$

   (e) $\displaystyle\int \frac{1}{\cos x - 2\sin x + 1}\,dx$