Contents

Chapter 10

The Definite Integral

10.1 The Definite Integral and Area

In problems 1 to 19 do the following:

(i) Graph the function.

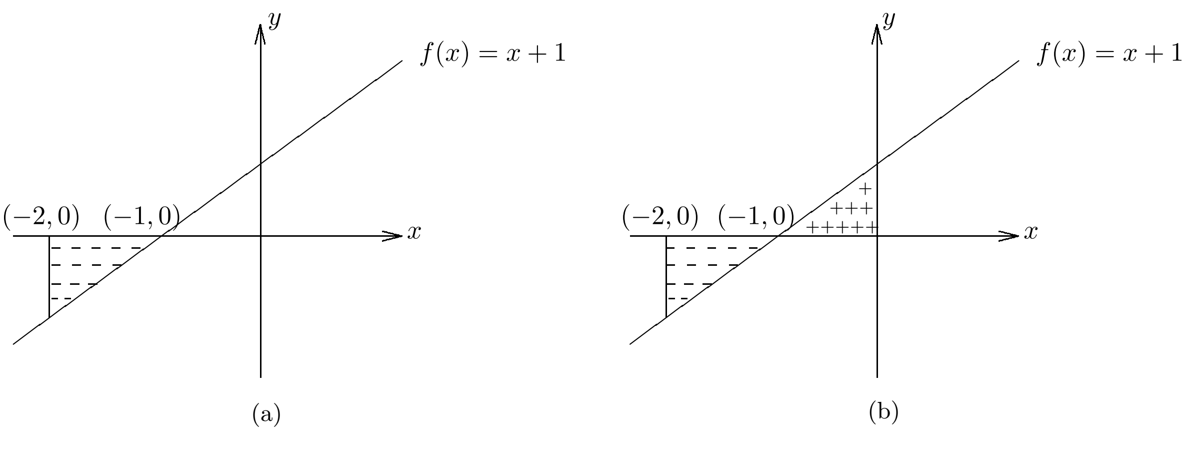
(ii) Shade in the area between the function and the interval of integration on the x-axis with plus or minus signs depending whether the given area is above and/or below the x-axis. *Draw a separate diagram for each interval of integration.*

(iii) Compute the definite integral.

1. $f(x) = x + 1$

(a) $\displaystyle\int_{-2}^{-1} f(x)dx$ (b) $\displaystyle\int_{-2}^{0} f(d)dx$

Solution for 1:



1. (a) $\displaystyle\int_{-2}^{-1} f(x)dx = -\frac{1}{2}$

   (b) $\displaystyle\int_{-2}^{0} f(x)dx = \int_{-2}^{-1} f(x)dx + \int_{-1}^{0} f(x)dx$
   $= -\frac{1}{2} + \frac{1}{2} = 0$

2. $f(x) = x + 1$.

   (a) $\displaystyle\int_{-1}^{9} f(x)dx$

   (b) $\displaystyle\int_{-1}^{1} f(x)dx$

   (c) $\displaystyle\int_{-2}^{1} f(x)dx$

3. (a) $\displaystyle\int_{-1}^{0} (|x-2|-x)dx$

   (b) $\displaystyle\int_{-1}^{1} (|x-2|-x)dx$

4. (a) $\displaystyle\int_{0}^{2} (|x-2|-x)dx$

   (b) $\displaystyle\int_{1}^{5} (|x-2|-x)dx$

5. (a) $\displaystyle\int_{1}^{5} (|x-2|-x)dx - \int_{0}^{1} (|x-2|-x)dx$

   (b) $\displaystyle\int_{0}^{1} (|x-2|-x) - \int_{1}^{5} (|x-2|-x)dx$

6. Let $h(x) = |x-2| - 2$

   (a) $\displaystyle\int_{-2}^{2} h(x)dx$

   (b) $\displaystyle\int_{-2}^{6} h(x)dx$

7. Let $n(x) = |x-2| - 2$

   (a) $\displaystyle\int_{0}^{4} n(x)dx$

   (b) $\displaystyle\int_{-2}^{0} n(x)dx - \int_{0}^{4} n(x)dx + \int_{4}^{6} n(x)dx$

   (Do the three integrals separately.)

8. (a) $\displaystyle\int_{0}^{2} |x-2|dx - 2\int_{0}^{2} dx$

   (b) $\displaystyle\int_{0}^{4} |x-2|dx + \int_{4}^{0} 2dx$

9. $C(x) = \sqrt{1-(x-1)^2}$

   (a) $\displaystyle\int_{0}^{2} C(x)dx$

   (b) $\displaystyle\int_{1}^{2} C(x)dx$

10. $G(x) = \begin{cases} \sqrt{1-(x-1)^2}, & x \in [0,2] \\ -\sqrt{1-(x+1)^2}, & x \in [-2,0) \end{cases}$

    (a) $\displaystyle\int_{-2}^{2} G(x)dx$

    (b) $\displaystyle\int_{-1}^{2} G(x)dx$

11. Let $F(x) = x$

    (a) $\displaystyle\int_{0}^{2} F(x)dx$

    (b) $\displaystyle\int_{2}^{4} F(x-2)dx$

12. Let $A(x) = |x|$

(a) $\displaystyle\int_{-1}^{0} A(x)dx$ (b) $\displaystyle\int_{9}^{10} A(x-10)dx$

13. Let $f(x) = x + 1$

(a) $\displaystyle\int_{-2}^{-1} f(x)dx$ (b) $\displaystyle\int_{2}^{3} f(x-4)dx$

14. Let $L(x) = -x + 2$

(a) $\displaystyle\int_{2}^{4} L(x)dx$ (b) $\displaystyle\int_{5}^{7} L(x-3)dx$

15. Let $L(x) = -x + 2$

(a) $\frac{1}{2}\displaystyle\int_{1}^{2} L\left(\frac{x}{2}\right) dx$ (b) $4\displaystyle\int_{\frac{1}{2}}^{1} L(4x)dx$

16. Let $A(x) = |x|$

(a) $\displaystyle\int_{-3}^{6} A(x)dx$ (b) $3\displaystyle\int_{-1}^{2} A(3x)dx$

17. Let $f(x) = \frac{1}{2}$. Compute the following:

$$\int_{1}^{2} f(x)dx, \quad \int_{1}^{2} 4f(x)dx, \quad \int_{-3}^{-4} f(x)dx, \quad \int_{41}^{42} f(x-40)dx, \quad \int_{23}^{24} f(x+20)dx$$

18. Let $g(x) = 2x - 4$

(a) $\displaystyle\int_{0}^{2} g(x)dx$ (b) $\displaystyle\int_{2}^{4} g(x)dx$

19. Let $S(x) = \begin{cases} \dfrac{3x}{4}, & x \in [0,4] \\ \sqrt{25 - x^2}, & x \in (4,5] \end{cases}$

(a) $\displaystyle\int_{0}^{5} S(x)dx$ (b) $\displaystyle\int_{0}^{4} S(x)dx$ (c) $\displaystyle\int_{4}^{5} S(x)dx$

In problems 20 to 39 give the value of the definite integral and explain in words and/or a diagram how you got your answer. If you use the fact that the function is even or odd you show this is true.

20. If $f(x)$ is integrable on $[-1,1]$, $f(x)$ is an *even* function, and $\displaystyle\int_{0}^{1} f(x)dx = \frac{1}{3}$

Find $\displaystyle\int_{-1}^{1} f(x)dx$ and $\displaystyle\int_{-1}^{0} 3f(x)dx$

21. If $f(x)$ is integrable on $[-\pi, \pi]$, $f(x)$ is an odd function and $\int_0^{\pi} f(x)dx = \frac{1}{4}$

    Find $\int_{-\pi}^{\pi} f(x)dx$

22. $\int_0^2 |x|dx = 2$, find $\int_{-2}^0 |x|dx$ and $\int_{-2}^2 |x|dx$

23. $\int_2^{-2} \frac{x^3}{1-x^2}\,dx$

24. $\int_{-1}^1 (t^3+t)dt$

25. $\int_{-\pi}^{\pi} x^2 \sin x\,dx$

26. $\int_{-4}^4 x^{15} \cos x\,dx$

27. $\int_{-\pi}^{\pi} \sin x \cos x\,dx$

28. $\int_{-\pi}^{\pi} \sin 2x\,dx$

29. If $g(x) = 2x - 4$ evaluate

    $\int_0^2 g(x)dx,\qquad \int_0^4 g(x)dx,\qquad \int_2^4 g(x)dx,\qquad \int_5^7 g(x-3)dx,\qquad \frac{1}{2}\int_4^8 g\left(\frac{1}{2}\right)dx$

30. If $f(x) = 3x$ evaluate

    $\int_0^1 (f(x)-3)dx,\qquad \int_1^3 (f(x)-3)dx,\qquad \int_0^{-1} (f(x)-3)dx$

31. $\int_0^1 \sqrt{1-x^2}\,dx,\qquad \int_0^{-1} \sqrt{1-x^2}\,dx,\qquad \int_{-1}^0 \sqrt{1-x^2}\,dx,\qquad \int_1^0 \sqrt{1-x^2}\,dx$

32. If $f(x) = 2$ and $\int_0^a f(x)dx = 1$, find a

33. If $g(x) = \pi$ and $\int_{-b}^b g(x)dx = 1$, find b

34. If $g(x) = \frac{x}{2}$ and $\int_2^b g(x)dx = 1$, find b

35. $\int_{-1}^1 |x|dx = ?$

36. If $f(x) = c$, $c \neq 0$ evaluate $\int_0^{\frac{1}{c}} f(x)dx$

37. If $g(x) = \frac{1}{a}$, $a \neq 0$ evaluate $\int_0^a g(x)dx$

38. If $g(x) = \frac{1}{a}$, $a \neq 0$ evaluate $\int_0^{a^2} g(x)dx$

39. If $h(x) = x + |x+2|$ evaluate
$$\int_{-2}^{-1} h(x)dx, \qquad \int_{-1}^{0} h(x)dx, \qquad \int_{-3}^{-2} h(x)dx, \qquad \int_{0}^{1} h(x)dx$$

In problems 40 to 42 assume f is integrable on $[1,7]$ and that $\int_1^5 f(t)dt = 3$, $\int_2^3 f(\theta)d\theta = 1$, $\int_3^5 f(x)dx = 1$, and $\int_3^7 f(y)dy = 6$. Evaluate:

40. $\int_1^3 f(x)dx$ and $\int_2^5 f(x)dx$

41. $\int_2^7 f(x)dx$ and $\int_5^7 f(x)dx$

42. $\int_1^2 f(x)dx$ and $\int_1^7 f(x)dx$

In problem 43 to 45 assume g is integrable on $[1,9]$ and that $\int_1^4 g(x)dx = 1$, $\int_2^4 g(r)dr = 2$, $\int_2^6 g(s)ds = 0$, $\int_6^9 g(t)dt = 1$. Evaluate:

43. $\int_1^2 g(u)du$ and $\int_2^9 g(v)dv$

44. $\int_4^6 g(w)dw$ and $\int_1^9 g(x)dx$

45. $\int_4^9 g(y)dy$ and $\int_1^6 g(z)dz$

10.2 Integrals with Variable Limits

In problems 46 to 61 do the following:

(i) Express the function, F, defined by the rule $F(x) = \int_a^x f(t)dt$, as a closed algebraic and/or trigonometric form; i.e., without the integral. (This can be done for *these* problems although many functions defined by an integral with variable upper limit cannot be expressed in this way.)

(ii) Sketch the graph of $f(x)$ and shade in area under the curve for the appropriate interval.

(iii) Sketch the graph of $F(x)$.

(iv) Compute $F(z)$ for given values of z.

46. (a) $F(x) = \int_1^x 2dt, \; x \in \mathbb{R}; \; z = 0, 1$

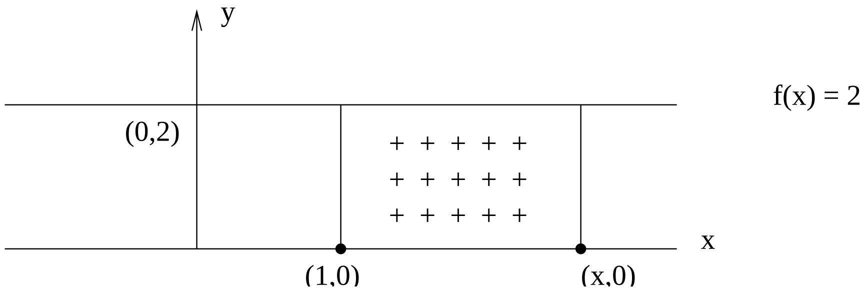
(b) $G(x) = \int_x^{x^2} 2dt, \; x \in \mathbb{R}; \; z = 0, 1, \frac{1}{2}, 2$

Solution for 46:

(a) (i) $f(x) = 2$ for every $x \in \mathbb{R}$

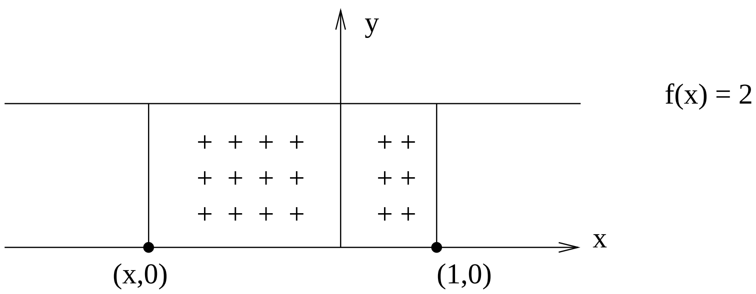
$\therefore F(x) = 2x - 2 = 2(x-1)$

(ii) *Case 1* $x > 1$



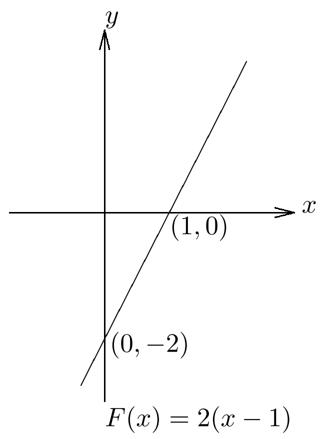
Case 2 $x < 1$

$$\int_1^x 2dt = -\int_x^1 2dt$$



where $\int_x^1 2dt$ is the indicated area.

(iii)



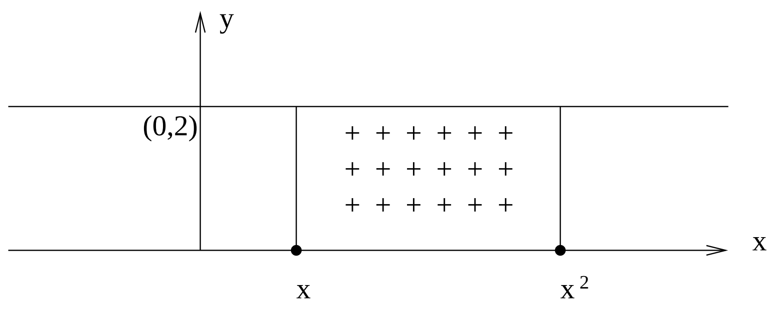
(iv) $F(0) = \int_1^0 2dt = 2(0-1) = -2$

$F(1) = \int_1^1 2dt = 2(1-1) = 0$

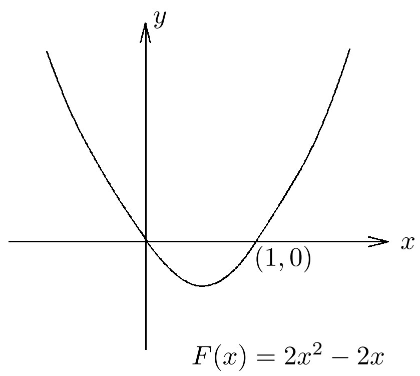
(b) (i) $\int_x^{x^2} 2dt = \int_x^0 2dt + \int_0^{x^2} 2dt$

$= \int_0^{x^2} 2dt - \int_0^x 2dt = 2x^2 - 2x$

(ii) $f(x) = 2$



(iii)



(iv) $F(0) = \int_0^0 2dt = 0$

$F(1) = \int_1^1 2dt = 0$

$F(\frac{1}{2}) = \int_{\frac{1}{2}}^{\frac{1}{4}} 2dt = -\int_{\frac{1}{4}}^{\frac{1}{2}} 2dt = 2 \cdot (\frac{1}{2})^2 - 2 \cdot \frac{1}{2} = -\frac{1}{2}$

$F(2) = \int_2^4 2dt = 4 = 2 \cdot (2)^2 - 2 \cdot 2$

47. $F(x) = \int_0^x f(t)dt$, where $f(x) = -3$, $x \in \mathbb{R}$ and $z = -1, 0, 1, 2$

48. $F(x) = \int_{\sqrt{x}}^x f(t)dt$, where $f(x) = -3$, $x > 0$ and $z = 0, 1, 16$

49. $F(x) = \int_0^x A(t)dt$, where $A(x) = |x|$, $x \in \mathbb{R}$ and $z = -3, -1, 0, 1, 2$

50. $F(x) = \int_1^x A(t)$, where $A(x) = |x|$, $x \in \mathbb{R}$ and $z = -3, -1, 0, 1, 2$

51. $F(x) = \int_0^x L(t)dt$, where $L(x) = 4x - 2$, $x \in \mathbb{R}$ and $z = -1, 0, 1$

52. $F(x) = \int_0^x L(t)dt$, $L(x) = x - 1$, $x \in \mathbb{R}$ and $z = -1, 0, 1$

53. $F(x) = \int_{g(x)}^{h(x)} L(t)dt$ where $L(x) = 2x - 1$; $h(x) = \frac{x^2}{2}$; $g(x) = x$

54. $F(x) = \int_1^{\sqrt{x}} t\,dt$, $x \geq 0$, $z = 0, 1, 4$

55. $F(x) = \int_\pi^x \sin t\,dt$, $z = 0, \frac{\pi}{2}, \pi, -\pi$

56. $F(x) = \int_0^{2x} \cos t\,dt,\ z = 0, \frac{\pi}{4}, \frac{\pi}{2}, \pi$

57. $F(x) = \int_0^{\sin x} dt,\ z = 0, \frac{\pi}{2}, \pi$

58. $F(x) = \int_{-\cos x}^{\sin x} dt,\ z = -1, 0, \frac{\pi}{2}$

59. $F(x) = \int_x^{x+\pi} \cos t\,dt,\ z = 0, \frac{\pi}{2}, \pi$

60. $F(x) = \int_{-x}^{x} t^2 dt$

61. $F(x) = \int_{|x|}^{x} dt$

10.3 Derivative of an Integral

In problems 62 to 73 find the following properties of $F(x)$ defined by an integral. (Do not attempt to first express $F(x)$ in a form without the integral):

(i) The x and y intercept if they exist.

(ii) The derivatives $F'(x)$ and $F''(x)$.

(iii) The intervals on which $F(x)$ is an increasing function and the intervals on which $F(x)$ is a decreasing function.

(iv) Relative maximum and minimum points if they exist.

(v) Where the function is concave up or concave down.

(vi) Any inflection points if they exist.

62. $F(x) = \int_{-1}^{x} (t^2 - t)dt,\ x \in \mathbb{R}$. Let $F(x) = \int_{-1}^{x} f(t)dt$.

Solution:

(i) $F(0) = \int_{-1}^{0} (t^2 - t)dt$

Therefore the y intercept is $\left(0, \int_{-1}^{0} f(t)dt\right)$

$F(-1) = \int_{-1}^{-1} f(t)dt = 0$

Therefore the x intercept is $(-1, 0)$

(ii) $F'(x) = x^2 - x = x(x-1)$

$F''(x) = 2x - 1$

(iii) $F'(x) > 0$ for $x < 0$ and $x > 1$
Therefore $F(x)$ is an increasing function in the intervals $(-\infty, 0)$ and $(1, \infty)$.
$F'(x) < 0$ for $x \in (0, 1)$
Therefore $F(x)$ is a decreasing function in this interval

(iv) $F'(x) = 0$ for $x = 0$ and $x = 1$
$x = 0 \Rightarrow F''(0) = -1 \Rightarrow F''(0) < 0$
Therefore there is a relative maximum at $\left(0, \int_{-1}^{0} f(t)dt\right)$
$x = 1 \Rightarrow F''(1) = 1 \Rightarrow F''(1) > 0$
Therefore there is a relative minimum at $\left(1, \int_{-1}^{1} f(t)dt\right)$

(v) (a) $F''(x) > 0$ for all $x \in (\frac{1}{2}, \infty)$

Therefore concave up on $(\frac{1}{2}, \infty)$

(b) $F''(x) < 0$ for all $x \in (-\infty, \frac{1}{2})$

Therefore concave down on $(-\infty, \frac{1}{2})$

(c) Since $f(t) = t^2 - t$ is continuous on $\mathbb{R}$, $\int_{-1}^{x} f(t)dt$ exists for all real numbers x. In particular $F(\frac{1}{2})$ is defined.

(vi) (a), (b) and (c) $\Rightarrow$ $\underline{x = \frac{1}{2} \text{ is a point of inflection which occurs at}}$ $\left(\frac{1}{2}, \int_{-1}^{\frac{1}{2}} f(t)dt\right)$

63. $F(x) = \int_{0}^{x} t^2 dt,\ x \in \mathbb{R}$

64. $F(x) = \int_{0}^{x} (t^2 + 1)dt,\ x \in \mathbb{R}$

65. $F(x) = \int_{1}^{x} t^2 dt,\ x \in [1, \infty]$

66. $F(x) = \int_{x}^{0} (t + 4)dt,\ x \in [0, \infty)$

67. $F(x) = \int_{x}^{-1} (4t + 2)dt,\ x \in \mathbb{R}$

68. $F(x) = \int_{0}^{x} \sin t\, dt,\ x \in \mathbb{R}$

69. $F(x) = \int_{0}^{x} (\sin^2 t + \cos^2 t)dt,\ x \in \mathbb{R}$

70. $F(x) = \int_{0}^{x} \cos t\, dt,\ x \in \mathbb{R}$

71. $F(x) = \int_{x}^{\frac{\pi}{2}} \sin t\, dt,\ x \in \mathbb{R}$

72. $F(x) = \int_1^x \frac{dt}{t}$, $x > 0$

73. $F(x) = \int_1^x \frac{dt}{t^2}$, $x > 0$

In problems 74 to 87 find $F'(x)$.

74. $F(x) = \int_0^x \sqrt{t^2+1}\, dt$

75. $F(x) = \int_0^x \frac{dw}{w+1}$, $x > -1$

76. $F(x) = \int_x^1 \sqrt{t^4+1}\, dt$

77. $F(x) = \int_x^1 \frac{d\theta}{\theta+2}$

78. $F(x) = \int_{\sin x}^x t^2 dt$

Solution for 78. Let $G(x) = \int_0^x t^2 dt$. Then

$F(x) = \int_{\sin x}^x t^2 dt = \int_0^x t^2 dt + \int_{\sin x}^0 t^2 dt = \int_0^x t^2 dt - \int_0^{\sin x} t^2 dt = G(x) - G(\sin x)$

$F'(x) = \big(G(x) - G(\sin x)\big)'$

$= G'(x) - \big(G(\sin x)\big)' = G'(x) - G'(\sin x)\cos x$ (by chain rule)

$= x^2 - \sin^2 x \cos x$ (by fundamental theorem of calculus)

79. $F(x) = \int_1^{3x^2} (2z-3)dz$

80. $F(x) = \int_0^{x^2} t^{\frac{1}{3}}\, dt$

81. $F(x) = \int_\pi^{\sin x} \theta d\theta$

82. $F(x) = \int_0^{\sqrt{x}} t^2 dt$, $x \geq 0$

83. $F(x) = \int_{\sqrt{x}}^x (t+1)dt$

84. $F(x) = \int_{x^2}^{x^3} \frac{1}{1+t^3}\, dt$

85. $F(x) = \int_{3x-2}^{x^2} y\sqrt{y+1}\, dy$

86. $F(x) = \int_x^{x^2} (z-a)^3 dz$

87. $F(x) = \int_{x-1}^{x+1} (2t-1)dt$

In problems 88 to 97 find $f(x)$ and $f(1)$

88. $\int_{-1}^{x} f(t)dt = x^2 - x - 2$

Solution:

Let $F(x) = \int_{-1}^{x} f(t)dt$. Hence $F(x) = x^2 - x - 2 \Rightarrow F'(x) = 2x - 1$ but $F'(x) = f(x)$. Therefore $f(x) = F'(x) = 2x - 1$, and $f(1) = 1$

89. $\int_5^x f(t)dt = x^3 - 6x^2 + 25$

90. $\int_{-\frac{1}{2}}^{x} f(t)dt = 2x + 1$

91. $\int_0^x f(t)dt = x\cos(x-1)$

92. $\int_x^1 f(t)dt = x^3 - 1$

93. $\int_x^{-\pi} f(t)dt = \sin x \cos x$

94. $\int_{-5}^{x} f(t)dt = (x+5)\cos x$

95. $\int_0^x f(t)dt = x^2(1+x)$

96. $\int_x^{-1} f(t)dt = -\sin(x^2-1)$

97. $\int_4^x f(t)dt = \cos^2(x-4)$

In problems 98 to 103 find a function f and a value c such that:

98. $\displaystyle\int_c^x f(t)dt = \cos x - \frac{1}{2}$

99. $\displaystyle\int_c^x f(t)dt = x^2 - 2x + 1$

100. $\displaystyle\int_c^x f(t)dt = x(1 - x^2)$

101. $\displaystyle\int_x^c f(t)dt = \sin x$

102. $\displaystyle\int_c^x f(t)dt = x^3 - 1$

103. $\displaystyle\int_c^x f(t)dt = x^2 - 2x - 3$

In the following problems find $f''(x)$:

104. $\displaystyle f(x) = \int_1^x \left[\int_1^t \frac{dy}{y}\right] dt$

105. $\displaystyle f(x) = \int_0^x \left[\int_1^t |y| dy\right] dt$

106. $\displaystyle f(x) = \int_1^x \left[\int_0^t \sin\theta^2 d\theta\right] dt$

107. $\displaystyle f(x) = \int_{-1}^x \left[\int_t^{t^2} d\theta\right] dt$

108. $\displaystyle f(x) = \int_0^x \left[\int_1^t \sqrt{y^3+1}\, dy\right] dt$

10.4 Integrals of Derivatives

Evaluate the following integrals:

109. $\displaystyle\int_0^1 (x+1)' dx$

110. $\displaystyle\int_{-1}^1 (x^2 - 1)' dx$

111. $\displaystyle\int_{-\pi}^{\pi} (x^3 - 1)' dx$

112. $\displaystyle\int_{-1}^0 \left(\sqrt{x+1}\right)' dx$

113. $\displaystyle\int_1^2 \left(\frac{1}{x}\right)' dx$

114. $\displaystyle\int_0^{\pi} (\sin\theta)' d\theta$

115. $\displaystyle\int_0^{\frac{\pi}{4}} (\tan\theta)' d\theta$

116. $\displaystyle\int_{-\pi}^{\pi} (\sin x \cos x)' dx$

117. $\displaystyle\int_0^x (t^2 - 1)' dt$

118. $\displaystyle\int_{-x}^x (t^3 + 1)' dt$

119. $\displaystyle\int_{-x}^x (t^2 + 9)' dt$

120. $\displaystyle\int_{x-1}^{x+1} (2t^2 + 1)' dt$

121. $\displaystyle\int_{\sqrt{x}}^x (\sin\theta^2)' d\theta$

122. $\displaystyle\int_{\cos\theta}^{\sin\theta} (t^2)' dt$

Evaluate the following integrals:

123. $\int_0^1 g''(t)dt$, where $g(t) = \int_1^t \sqrt{x^2+1}\,dx$

124. $\int_{-1}^1 g''(t)dt$, where $g(t) = \int_1^t (x^4-16)dx$

125. $\int_0^\pi g''(t)dt$, where $g(t) = \int_1^t \sin\theta\,d\theta$

126. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} g''(t)dt$, where $g(t) = \int_1^t \sin\theta\,d\theta$

127. $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} g''(t)dt$, where $g(t) = \int_1^t \cos\theta\,d\theta$

128. $\int_3^2 g''(t)dt$, where $g(t) = \int_1^t \frac{dx}{x}$

10.5 Evaluation of Definite Integrals

Evaluate the following integrals and include the intermediate steps in your solution.

129. $\int_{-2}^3 x\,dx$

Solution of 129. *Let* $F_1(x) = x^2$,
$F_1'(x) = 2x$ Therefore try:
$F(x) = \frac{x^2}{2}$, $F'(x) = x$
Hence $\int_{-2}^3 x\,dx = F(3) - F(-2)$
$F(3) - F(-2) = \frac{9}{2} - \frac{4}{2} = \frac{5}{2}$.

130. $\int_3^1 2x\,dx$

131. $\int_{-3}^2 x^2 dx$

132. $\int_9^{11} (t-9)^2 dt$

133. $\int_0^1 x^4 dx$

134. $\int_4^9 y\sqrt{y}\,dy$

135. $\int_2^5 \left(u + \frac{1}{u^2}\right) du$

136. $\int_1^4 \frac{dy}{y^2}$

137. $\int_1^3 \frac{ds}{\sqrt{s}}$

138. $\int_{-3}^2 (3x+2x+1)dx$

139. $\int_1^3 (z^3+2)dz$

140. $\int_1^2 (x^2 + x + 1 + x^{-2})dx$

141. $\int_0^1 (x+2)^2 dx$

142. $\int_{-2}^{-1} \left(x^{31} - 3x^{\frac{1}{3}}\right) dx$

143. $\int_{-7}^{7} 16(x^2+1)2x\,dx$

144. $\int_1^3 x^{\frac{2}{3}} dx$

145. $\int_1^2 \frac{x^3-1}{x-1}\,dx$

146. $\int_0^{\pi} \sin x\,dx$

147. $\int_0^{\frac{\pi}{4}} \cos x\,dx$

148. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2\theta\,d\theta$

149. $\int_{\pi}^{-\pi} (x + 2\cos x)dx$

150. $\int_0^{\frac{\pi}{2}} (\cos x + \sin x)dx$

151. $\int_0^{\frac{\pi}{4}} (1 + \tan^2)dy$

In problems 152 to 161 find the following:

(i) the domain of the function;

(ii) the derivative of the function;

(iii) x such that $F(x) = 0$.

152. $F(x) = \int_1^{x+1} \frac{dt}{t}$

Solution:

(i) Let $f(t) = \frac{1}{t}$. $f(t)$ is continuous on the interval $(0, \infty)$ hence $\int_1^z f(t)dt$ exists for $z > 0$.
(Note if $x < 0$ then $0 \in [z, 1]$, $f(t)$ blows up as $t \to 0$, and $\int_1^z f(t)dt$ does not exist on this interval.)
Let $z = x+1$; $z > 0$ implies $x+1 > 0$ implies $x > -1$ therefore the domain of $F(x)$ is all $x > -1$ or $x \in (-1, \infty)$.

(ii) $\ln x = \int_1^x \frac{dt}{t}$, so $F(x) = \int_1^{x+1} \frac{dt}{t} = \ln(x+1)$
$F'(x) = \big(\ln(x+1)\big)' = [\ln'(x+1)](x+1)' = \frac{1}{x+1}$.

(iii) If $F(x) = 0$ then $\ln(x+1) = 0$, and so $x+1 = 1$; i.e., $x = 0$.

153. $F(x) = \int_1^{3x} \frac{dt}{t}$

154. $F(x) = \int_1^{x^2} \frac{dt}{t}$

155. $F(x) = \int_1^{2x-1} \frac{dt}{t}$

156. $F(x) = \int_1^{\cos x} \frac{dt}{t}$

157. $F(x) = \int_1^{\frac{1}{x}} \frac{dt}{t}$

158. $F(x) = \int_1^{1+\cos x} \frac{dt}{t}$

159. $F(x)=\int_1^{2+\sin x}\frac{dt}{t}$

160. $F(x)=\int_0^{x-1}\frac{dt}{t+1}$

161. $F(x)=\int_{-2}^{x-3}\frac{dt}{t+1}$

162. $F(x)=\int_2^{x+1}\frac{dt}{t-1}$

In problems 163 to 166 evaluate the given integral.

163. $\int_0^4 e^x dx$

164. $\int_0^1 4e^x dx$

165. $\int_0^1 e^2 dx$

166. $\int_0^1 e^3 e^{-3} dx$

167. $\int_{-x}^{x} e^{\pi} dt$

168. $\int_{-\pi}^{\pi} e^3 \sin x \, dx$

169. $\int_0^1 \frac{e^x + e^{-x}}{2} dx$

170. $\int_0^1 \frac{e^x - e^{-x}}{2} dx$

171. $\int_0^x e^{\sin\pi} t \, dt$

172. $\int_0^1 \exp(1) dx$

173. $\int_{-1}^1 \exp(0) x^3 dx$

174. $\int_x^0 \exp(2)\exp(4) t^2 dt$

175. $\int_2^4 \exp(3+x) dx$

176. $\int_1^4 [1+\exp(x)] dx$

177. $\int_{-\pi}^{\pi} [\exp(x)+\sin x] dx$

178. $\int_0^x \exp\left(\sin\frac{\pi}{2}\right) dt$

179. $\int_{-1}^1 \exp\left(\cos\frac{\pi}{2}\right)\sin x \, dx$