Contents

Chapter 15

Differential Equations

15.1 Simple Differential Equations

Verify that the differential equation in question 1 to 16 has the given function as a solution.

1. $x\frac{dy}{dx} + y = x^2; \quad f(x) = \frac{1}{3}x^2 + \frac{c}{x}$ NOTE: $y = f(x)$ and $f'(x) = \frac{dy}{dx}$

Solution:

The derivative of f is $f'(x) = \frac{2x}{3} - \frac{c}{x^2}$. Substitute in for $\frac{dy}{dx}$

$$x\left(\frac{2}{3}x - \frac{c}{x^2}\right) + \left(\frac{1}{3}x^2 + \frac{c}{x}\right) = x^2.$$

Equation is valid for all x for which $f(x)$ is defined, namely $x \neq 0$.

2. $\frac{dy}{dx} = x^2; \quad f(x) = \frac{1}{3}x^3$

3. $\frac{dy}{dx} = 3x + 5; \quad f(x) = \frac{3}{2}x^2 + 5x - 2$

4. $\frac{ds}{dt} = \frac{1}{\sqrt{t-1}}; \quad f(t) = 2(\sqrt{t-1} + 1)$

5. $x^3y' = x^4 - \sqrt{3}; \quad g(x) = \frac{1}{2}x^2 + \frac{\sqrt{3}}{2x^2}$

6. $\frac{d^2y}{dx^2} = x, y = \frac{1}{6}(x^3 + 2x - 2)$

7. $\frac{d^2s}{dt^2} - 2\frac{ds}{dt} + 10 = 0; \quad s = 5t$

8. $xy' = 2y; \quad y = Cx^2$

9. $y''' - y'' = 0; \quad y = C_1 + C_2x$

10. $y\frac{dy}{dx} = x^2, \quad y = \sqrt{\frac{2}{3}x^3 + 4}$

11. $\frac{u}{x}\ \frac{du}{dx} = 1; \quad u = \sqrt{x^2 + k}$

12. $\frac{dy}{dx} = \frac{y^2}{x^2}; \quad y = \frac{x}{1-ax}$

13. $x(y')^2 - yy' + 1 = 0; \quad y = C + \frac{x}{C}$

14. $\frac{dx}{dt} + \frac{x}{t} = \sqrt{t^2+1}; \quad x = \frac{(t^2+1)^{\frac{3}{2}}+C}{3t}$

15. $D_x^2 y - 3xD_x y + 3y = 2 - 3x^2; \quad y = x^2 + 3x$

16. $xD_x^2 y - xD_x y = 12x^2 - 6x^3; \quad y = 2x^3 + 5$

In problems numbered 17 to 40, find the solution of the differential equation satisfying the initial conditions.

17. $y' = 2x - 4; \quad y = 3$ at $x = 3$

Solution:

$y = \int 2x - 4dx$ or $y = x^2 - 4x + C$ where C is an arbitrary constant.
To determine C substitute in the initial conditions $(x = 3,\ y = 3)$

$$3 = 9 - 12 + C \quad \text{or} \quad C = 6.$$

The answer is $y = x^2 - 4x + 6$.

Comment: $y = x^2 - 4x + C$ is the general solution.
$y = x^2 - 4x + 6$ is the particular solution.

18. $y' = 4x + 7; \quad y = 3$ at $x = 2$

19. $y' = -x^2 - 3; \quad y = 4$ at $x = -1$

20. $\frac{ds}{dt} = 3t - 5; \quad s = -3$ at $t = 2$

21. $F'(t) = 1 - 6t + t^2; \quad F(-1) = 4$ $\left[F(t) = \frac{t}{3} - 3t + t + \frac{25}{3}\right]$

22. $\frac{dv}{dt} = -2t + 3; \quad v = 4$ at $t = 3$

23. $\frac{dy}{dx} = \sqrt{7}x^3 + 3x - 2; \quad y = -5$ when $x = 0$

24. $y' = ax^2; \quad y = a$ when $x = 1$

25. $\frac{1}{t}\frac{dy}{dt} = 2t^2 + t + 1; \quad y = 5$ when $t = \frac{1}{2}$

26. $\frac{ds}{dt} = \sqrt{1-2t}; \quad s = 0$ when $t = -4$

27. $\frac{1}{x}\frac{dy}{dx} = 8\sqrt[3]{1+2x^2}; \quad y = 2$ when $x = 0$

28. $y' = \frac{-x}{\sqrt{10-x^2}}; \quad y = 3$ when $x = -2$

29. $\frac{d^2y}{dx^2} = 12x - 4; \quad y = 4$ and $y' = 3$ at $x = 2$

Solution:

$y' = \frac{dy}{dx}$ so that $\frac{dy'}{dx} = 12x - 4$ which is considered as a differential equation involving y' as a function of x with initial conditions $(y' = 3$ at $x = 2)$

$y' = \int(12x - 4)dx$ or $y' = 6x^2 - 4x + C_1$

When $x = 2$, $y' = 3$ so $C_1 = -13$ and $y' = 6x^2 - 4x - 13$.

$y = \int(6x^2 - 4x - 13)dx$ or $y = 2x^3 - 2x^2 - 13x + C_2$.

When $x = 2$, $y = 4$ so $4 = 16 - 8 - 26 + C_2$ or $C_2 = 22$

Solution to #29 (cont'd)

The particular solution is $y = 2x^3 - 2x^2 - 13x + 22$.

Comment: Without initial conditions C_1 and C_2 would remain arbitrary and the general solution for the above question would be $y = 2x^3 - 2x^2 + C_1x + C_2$.

30. $F''(t) = -4; \quad F(0) = 2,\ F'(0) = 1$

31. $f''(x) = x; \quad f(2) = \frac{1}{2},\ f'(2) = 1$

32. $\frac{d^2s}{dt^2} = g$ (g const); when $t = 0,\ s = 12$, and $\frac{ds}{dt} = 0$. $\left[y = \frac{1}{2}x^3 - x^2 - 18x + 60\right]$

33. $\frac{d^2y}{dx^2} = 3x - 2; \quad y = 4$ and $\frac{dy}{dx} = -2$ at $x = 4$

34. $\frac{d^y}{dx^2} = -5; \quad y = -1$ and $\frac{dy}{dx} = 3$ at $x = -2$

35. $\frac{d^2u}{dt^2} = -2t^2 + 5; \quad u = 4$ and $\frac{du}{dt} = -2$ at $t = -1$

36. $\frac{dy}{dx} = \sqrt{6x+1}; \quad y = 1$ when $x = \frac{1}{2}$

37. $\frac{d^2y}{dx^2} = \frac{1}{\sqrt{x}} + 1; \quad \frac{dy}{dx} = 2$ when $x = 4$, and $y = 0$ when $x = 1$ $\left[y = \frac{4}{3}x^{\frac{3}{2}} + \frac{1}{2}x^2 - 6x + \frac{25}{6}\right]$

38. $f''(x) = 2x; \quad f(1) = 2,\ f(-1) = -3$

39. $\frac{dy}{dx} = 4e^{2x} + 5e^{-x};$ and $y = 2$ at $x = 0$

40. $\frac{dx}{dt} = 5\log t + 2e^t;$ and $x = 3e$ at $t = 1$

Verify that the differential equations in questions numbered 41 to 48 have the indicted general solutions. Find the particular solution through the given point (a, b).

41. $\frac{dy}{dx} = x + 1; \quad y = \left(\frac{x+1}{2}\right)^2 + C; \quad a = -1,\ b = 0$

42. $\frac{dy}{dx} = kx; \quad y = Ae^{kx}; \quad a = 0,\ b = 1$

43. $\frac{dy}{dx} = \frac{1}{x}; \quad y = \ln(Ax); \quad a = 1,\ b = 0$

44. $\frac{dy}{dx} = \frac{y}{x}; \quad y = Ax; \quad a = 1,\ b = 1$

45. $\frac{dy}{dx} = \frac{y^2}{x^2}; \quad y = \frac{x}{xC+1}; \quad a = 1,\ b = 1$

46. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 0; \quad y = Ae^{-3x} + Be^{-x}; \quad a = 0,\ b = 1$

47. $\frac{d^2y}{dx^2} + 4y = 0; \quad y = A\sin(2x + B); \quad a = 0,\ b = 0$

48. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0; \quad y = (A + Bx)e^{-x}; \quad a = 0,\ b = 1$

Find the general solution of the following differential equations.

49. $f'(x) = \frac{1}{2}x$ $\left[y = \frac{1}{4}x^2 + C\right]$

50. $\frac{dy}{dx} = -x + 5$

51. $f'(x) = x^3 - 3x^2 + 10$

52. $y' = \frac{4}{\sqrt{x}} - x$

53. $\frac{ds}{dt} = \frac{t^4 - t}{2t^3}$ $\left[S = \frac{t^2}{4} + \frac{1}{2t} + C\right]$

54. $x^2 \, \frac{dy}{dx} = x^2 + 1$

55. $\sqrt{x} \, \frac{du}{dx} = (1 - x)^2$

56. $z' = \frac{3x}{\sqrt{1 - x^2}}$

57. $\frac{1}{t^2} g'(t) = \frac{-7}{(1+t^3)^2}$ $\left[g(t) = \frac{7}{3(1+t^3)} + C\right]$

58. $\frac{d^2y}{dx^2} = \frac{1}{x^3}$

59. $\frac{d^n y}{dx^n} = 0$

60. $\frac{d^3 s}{dt^3} = -5 + 4e^t$

Find the differential equation of lowest order satisfied by the functions in questions 61 to 72. (A, B, C, D are arbitrary constants.)

61. $y = \frac{1}{2}x^2 - x + C$

62. $y(x) = x^2 + Ax$

63. $y(x) = Be^x$

64. $y(x) = e^{x+B}$

65. $y(x) = \ln(Ax)$

66. $y(x) = Ax^2 + Bx + C$

67. $y(x) = \frac{x+A}{x+B}$

68. $y(x) = Ae^{2x} + Be^{-2x} + C$

69. $y(x) = (A + Bx)e^{3x}$

70. $y(x) = A\sin(Bx + D)$

71. $y(x) = A\cos(Bx + D)$

72. $y = Ae^{-\frac{1}{x}}$

15.2 Geometric Aspects of Differential Equations

In problems 73 to 88,

(a) obtain $f(x)$ so that $y = f(x)$ passes through the given point and has the specified slope;

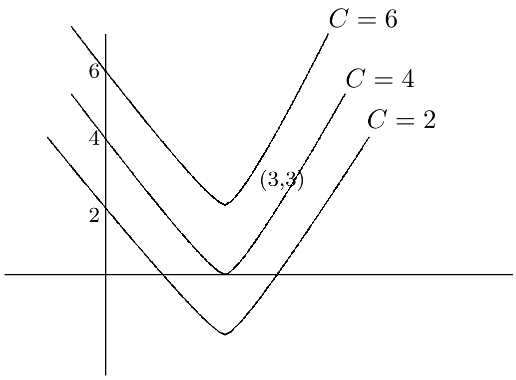
(b) graph the solution $y = f(x)$ and one other curve of the family determined by the differential equation which is used.

73. Slope $= 2x - 4$; through the point (3,3)

Solution: $y = \int(2x-4)dx = x^2 - 4x + C$

which is the general solution. It is a family of parabolas. At (3,3) $3 = 9 - 12 + C$, or $C = 6$.

Answer to part (a) is $y = x^2 - 4x + 6$.
Answer to part (b) is



74. Slope $= x - 2$; through the point (2,2)

75. Slope $= \theta - \theta^2$; through the point $(-1, 0)$

76. Slope $= 4x + 4$; through the point $(2, -12)$

77. Slope $= -6x - 12$; through the point $(1, -22)$ $\left[y = -3x^2 - 12x - 7\right]$

78. Slope $= 2/x$; through the point (1,3)

79. Slope $= -4/x^2$; through the point $(-2, -2)$

80. $\phi'(t) = 1 + 2t^2$; through the point $(4, -2)$

81. $\frac{dy}{dx} = 2 - x$; through the point (1,0) $\left[y = 2x - x^2 - \frac{3}{2}\right]$

82. $\frac{d\phi}{dt} = -t^2$; through the point (1,0)

83. Find an equation $y = g(x)$ of a curve such that $g''(x) = 3$ at each point (x, y) on it and passing through (2,1) with slope -1 there.

84. Find $f(x)$ so that $f''(x) = 3$ at all values of x, and the tangent to the graph of $y = f(x)$ at (2,1) is $y = 6x - 9$.

85. Find an equation $y = f(x)$ of a curve such that $f''(x) = 4x$ at each point (x, y) on it and crossing the x axis at $(-2, 0)$ at a 45° angle. $\left[y = \frac{1}{3}(2x^3 - 21x - 26)\right]$

86. Find an equation $y = f(x)$ of a curve such that $f''(x) = 3x^2$ at each point (x, y) on it and passing through $\left(1, \frac{9}{4}\right)$ and (2,5).

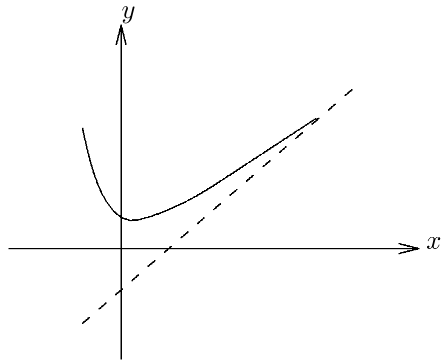
87. Find an equation $y = F(x)$ of a curve such that $F''(x) = 1/x^3$ at each point (x, y) on it and passing through (1,6).

88. Find $f(x)$ so that $f''(x) = -2$ at all values of x, and the tangent to the graph of $y = f(x)$ at $(-1, 2)$ is parallel to $4x - 2y = 5$.

In problems 89 to 104, sketch the direction field and draw a solution through the point indicated.

89. $\frac{dy}{dx} = x - y$

Solution :



90. $\frac{dy}{dx} = 2x; \quad (0,0)$

91. $\frac{dy}{dx} = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases}; \quad (1,1)$

92. $\frac{dy}{dx} = x^2; \quad (0,1)$

93. $\frac{dy}{dx} = y - 1; \quad (1,1)$

94. $\frac{dy}{dx} = \sqrt{y}; \quad (0,2)$

95. $y' = -\frac{y}{x}; \quad (1,1)$

96. $y' = \frac{y}{x}; \quad (1,1)$

97. $y' = \frac{x}{y}; \quad (2,1)$

98. $y' = 2x + y; \quad (0,1)$

99. $y' = y^2; \quad (0,1)$

100. $y' = y^2 + x; \quad (0,1)$

101. $y' = xy; \quad (0,1)$

102. $\frac{dy}{dx} = x^2 + y; \quad (0,0)$

103. $\frac{dy}{dx} = x^2 + y^2; \quad (1,1)$

104. $\frac{dy}{dx} = \frac{2x+y}{x-2y}; \quad (1,1)$

15.3 Solving Differential Equations

Variables Separable — Find solutions of the differential equations numbered 105 to 128.

105. $x^2 dy + y^2 dx = 0$

106. $x\,dy - y\,dx = 0$

107. $\frac{dy}{dx} = \frac{x-1}{y}$

108. $\frac{dy}{dx} = y^2$

109. $\frac{dy}{dx} = \frac{1+y}{1+x}$

110. $(1 - x^2)dy + (1 + y^2)dx = 0$

111. $\frac{ds}{dt} = \frac{t^2+3}{s^3}$

112. $(y + y^2)y' = x^2 - 3$

113. $y \sec x\,dy + \sqrt{1 - y^2}\,dx = 0$

114. $dy + y \tan x\,dx = 0$

115. $\frac{dy}{dx} = 3y^{\frac{2}{3}}$

116. $\frac{2}{y^2}\,\frac{dy}{dx} = x^2$

117. $\frac{dy}{dx} = \sqrt{\frac{1-x^2}{1-y^2}}$

118. $\sec x\,dy + \sec y\,dx = 0$

119. $\theta^2 r \frac{dr}{d\theta} = 1 + \theta^2$

120. $\frac{dy}{dx} = -\frac{y^2}{(x+a)^4}$

121. $xy\,dy + (x^2+1)dx = 0$

122. $\frac{dy}{dx} = \frac{x - xy^2}{x^2 y - y}$

123. $(1+x)\frac{dx}{dt} = tx^3$

124. $y' = \frac{3}{y + \sqrt{y}}$

125. $(xy - x)y' = y^2$ $\left[x = Aye^{\frac{1}{y}}\right]$

126. $y\sqrt{2x^2-3}\,\frac{dy}{dx} + x\sqrt{6+y^2} = 0$

127. $y' = \frac{2}{\sqrt{y'}}$

128. $\left(\frac{dy}{dx}\right)^2 = x^2$

For the differential equations in problems numbered 129 to 132, find a solution satisfying the given boundary condition.

129. $\frac{dy}{dx} = \frac{x^2}{y}$; $y = -3$ and $x = 0$

130. $\frac{dy}{dx} = y^2$; $y = 5$ when $x = 2$

131. $x^2(3+2y)y' = 1$; $y = 1$ when $x = -2$

132. $\frac{dy}{dx} = \frac{x}{(x^2-1)^2 y}$; $y = 5$ when $x = 0$

Problems

Set up the differential equation and solve the problem.

133. A bacterial population increases at a rate proportional to the population. If the population triples in one hour, in how many hours will it be 100 times its original size?

134. Radioactive carbon has a half-life of approximately 5600 years. In how many years will it decay to 20% of its original amount? To 10%?

135. The half-life of a certain radioactive isotope is 100 years. How long does it take to decay to 10 per cent of its initial amount?

136. A tank contains 100 gal. of pure water. Brine, containing 0.2 lb. of salt per gallon, runs into the tank at the rate of 2 gal/min, and the mixture runs out at the same rate. What is the amount of salt in the tank at the end of t min? 50 min?

137. If a certain radioactive substance loses 15% of its radioactivity in three days, what is its half-life?

138. In the above problem, how long will it take for the substance to be 95% dissipated?

139. A tank contains 100 gal. of brine containing 0.05 lb. of salt per gallon. Pure water begins to run in at the rate of 1 gal/min from one tap, brine containing 0.25 lb. of salt per gallon runs in from another tap at the rate of 2 gal/min, and the mixture runs out at the rate of 3 gal/min. What is the amount of salt in the tank at the end of t min?

140. If brakes on a car apply a constant deceleration of a ft/sec^2, find a formula for velocity and distance if initially the velocity is v_0 and the distance x_0.

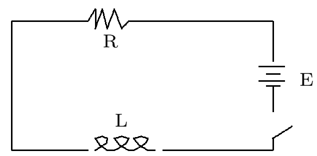
141. Since a thermometer is itself a physical body, it is subject to Newton's law of cooling. A thermometer is taken from inside a house, where the temperature is 80°, to the outdoors where the temperature is 40°. Five minutes later it reads 50°. What will it read 15 minutes after it is taken outdoors?

142. Newton's law of cooling states that the time rate of change of the temperature T of a body is proportional to the difference, $T - T_0$, between its temperature and that of its environment, T_0. If a body initially at 212°F (boiling water) cools to 152°F in 20 min. when the air is at 32°F, find the temperature after t min. of a body initially at 100°F.

143. In a simple series circuit with resistance R ohms, inductance L henrys and electromotive force E volts, the time rate of change of current i (in amperes) is

$$L\frac{di}{dt} + Ri = E,$$

where the time t is in seconds. Find the current at time t if no current is flowing when the switch is closed at $t = 0$.



144. A tank contains 50 gal. of brine containing 25 lb. of dissolved salt. If water runs in at the rate of 2 gal/min and the mixture runs out at the same rate, find the salt concentration after t minutes.

145. To what rate of interest, payable annually, is 6% interest, compounded continuously, equivalent?

146. A spherical drop of liquid evaporated at a rate proportional to its surface area. If a given drop evaporates to one-eighth its original volume in five minute,s in how many minutes will it evaporate completely?

147. A mathematics instructor finds the body of Professor Y murdered. He notes that the temperature of the professor's office is 21°C. He finds the temperature of the body to be 31°C. One hour later the body temperature is down to 29°C. Assuming the professor's body temperature was 37°C. When he died, find approximately how long after death the body was discovered.

P.S. Professor X who has taken over Professor Y's lectures insists this problem be a final exam question since the murderer will likely reveal himself by giving the correct answer to the problem without working it out using Newton's law of cooling.

15.4 Exact Equations

148. $d(xy) = 0$

149. $d(xy) + dy + dx = 0$

150. $d(xy) + y\,dy + x\,dx = 0$

151. $y\,dx + x\,dy = 0$

152. $(y + 4)dx + (x - 4)dy = 0$

153. $(y - 7)dx + (x + 10)dy = 0$

154. $(y + x)dx + (x - y)dy = 0$

155. $(y + x^3)dx + (x + \sqrt{y})dy = 0$

156. $(y + \sin x)dx + (x - \cos y)dy = 0$

157. $(y + e^x)dx + (x + e^y)dy = 0$

158. $\frac{x\,dy - y\,dx}{x^2} = 0$

159. $(3x^2y^2 + x^2)dx + (2x^3y + y^2)dy = 0$

160. $y \cos x\,dx + \sin x\,dy = 0$

161. $\cos x \cos y\,dx - \sin x \sin y\,dy = 0$

162. $\cos x \sin y\,dx + \sin x \cos y\,dy = 0$

163. $\sin x \cos y\,dx + \cos x \sin y\,dy = 0$

164. $\cos x\,dy - y \sin x\,dx = 0$

165. $2x \sin y\,dx + x^2 \cos y\,dy = 0$

166. $e^x e^y dx + e^x e^y dy = 0$

167. $ye^x dx + e^x dy = 0$

168. $(ye^x + y)dx + (e^x + x)dy = 0$

169. $2x - y + (y^2 - x)\frac{dy}{dx} = 0$

170. $\sin y + (x\cos y + y\cos y + \sin y)\frac{dy}{dx} = 0$

171. $2xy\,dx + (x^2 - y^2)dy = 0$

172. $(x + y)dx + (x + 2y)dy = 0$

173. $ye^x - x(e^x + 1)\frac{dy}{dx} = 0$

174. $(ye^{xy} + 2xy)dx + (xe^{xy} + x^2)dy = 0$

175. $(r + e^\theta)d\theta + (\theta + e^r)dr = 0$

176. $1 + r\cos\theta + \sin\theta\frac{dr}{d} = 0$

177. $ye^x - y + (e^x + 1)\frac{dy}{dx} = 0$

178. $y\sec^2 x\,dx + \tan x\,dy = 0$

179. $\ln(y^2 + 1) + \frac{2xy}{y^2+1}\ \frac{dy}{dx} = 0$

180. $(e^x \sin y + y)\frac{dy}{dx} = e^x \cos y$

15.5 Linear Equations of First Order

181. $y' + y = x$

182. $y' + y = e^{-x}$

183. $y' + y = e^x$

184. $2y' - y = e^{\frac{x}{2}}$

185. $y' + xy = x$

186. $y' - 2y = 3$

187. $y' + by = c$

188. $y' + 2y = x$

189. $xy' - 3y = x^2$

190. $xy' + 3y = \frac{\sin x}{x^2}$

191. $x\,dy + y\,dx = \sin x\,dx$

192. $x\,dy + y\,dx = y\,dy$

193. $y' - \frac{y}{x} = x\sin x$

194. $y' + y\tan x = \sin x$

195. $y' + y\tan x = \sec x$

196. $y' = e^{2x} + 3y$

197. $y' = e^{ax} + ay$

198. $y'\sin x + y\cos x = 1$

199. $y' - ay = f(x)$

200. $(x - 1)^3 y' + 4(x - 1)^2 y = x + 1$

201. $\cosh x\,dy + (y\sinh x + e^x)dx = 0$

202. $e^{2y}dx + 2(xe^{2y} - y)dy = 0$

203. $(x - 2y)dy + y\,dx = 0$

204. $(y^2 + 1)dx + (2xy + 1)dy = 0$

15.6 Homogeneous Linear Equations with Constant Coefficients

In problems 205 to 215 find the general solution.

Note when the operator D is used it is implied that the independent variable is x.

205. $(D^2 - D - 2)y = 0$

206. $(D^2 + 3D)y = 0$

207. $(D^2 - D - 6)y = 0$

208. $(D^2 + 5D + 6)y = 0$

209. $(D^3 + 2D^2 - 15D)y = 0$

210. $(D^3 + 2D^2 - 8D)y = 0$

211. $(D^3 - D^2 - 4D + 4)y = 0$

212. $(D^3 - 3D^2 - D + 3)y = 0$

213. $(4D^3 - 13D + 6)y = 0$

214. $(10D^3 + D^2 - 7D + 2)y = 0$

215. $\{D^2 - (a + b)D + ab\}y = 0, \quad a \neq b$

In problems 216 to 222 find the required particular solution.

216. $(D^2 - 2D - 3)y = 0;$ when $x = 0,\ y = 0$ and $y' = -4$

217. $(D^2 - D - 6)y = 0;$ when $x = 0,\ y = 0$ and when $x = 1,\ y = e^3$

218. $(D^2 - 2D - 3)y = 0;$ when $x = 0,\ y = 4$ and $y' = 0$

219. $(D^3 - 4D)y = 0;$ when $x = 0,\ y = 0,\ y' = 0$ and $y' = 2$

220. $(D^2 - D - 6)y = 0;$ when $x = 0,\ y = 3$ and $y' = -1$

221. $(D^2 + 3D - 10)y = 0;$ when $x = 0,\ y = 0$ and when $x = 2,\ y = 1$

222. $(D^3 - 2D^2 - 5D + 6)y = 0;$ when $x = 0,\ y = 1,\ y' = -7$ and $y' = -1$

In problems 223 to 249 find the general solution.

223. $(4D^2 - 4D + 1)y = 0$

224. $(D^2 + 6D + 9)y = 0$

225. $(D^3 - 4D^2 + 4D)y = 0$

226. $(9D^3 + 6D^2 + D)y = 0$

227. $(2D^4 - 3D^3 - 2D^2)y = 0$

228. $(2D^4 - 5D^3 - 3D^2)y = 0$

229. $(D^3 + 3D^2 - 4)y = 0$

230. $(4D^3 - 27D + 27)y = 0$

231. $(D^3 + 3D^2 + 3D + 1)y = 0$

232. $(D^3 + 6D^2 + 12D + 8)y = 0$

233. $(D^5 - D^3)y = 0$

234. $(D^5 - 16D^3)y = 0$

235. $y'' - y = 2$

236. $y'' - y = x$

237. $y'' - y = x + 1$

238. $y'' - y = x^2$

239. $y'' - 2y = 2$

240. $y'' - 4y = 1$

241. $y'' - y' = x$

242. $y'' - y' = x^2$

243. $y'' - y' = x^2 + 2x$

244. $y'' + y' = x$

245. $y'' - 5y' + 4y = x^2 - 2x + 1$

246. $y'' + y' - 6y = 2x^3 + 5x^2 - 7x + 2$

247. $y'' - 4y = x^3$

248. $y'' - 2y' - 3y = 4 - 8x - 6x^2$

249. $y'' - y' = x$

250. For the equation $(D^3 + D^2)y = 4$ find the solution whose graph has at the origin a point of inflection with a horizontal tangent line.

251. For the equation $(D^2 - D)y = 2 - 2x$ find a particular solution which has at some point (to be determined) on the x-axis an inflection point with a horizontal tangent line.