Contents

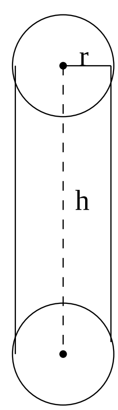
Chapter 4

Related Rates

Problem

If a tree trunk adds $1/4$ of an inch to its diameter and 1 foot to its height each year, how rapidly is its volume changing when its diameter is 3 feet and its height is 50 feet (assume that the tree trunk is a circular cylinder).

Solution



Let r be the radius and h be the height of the trunk. Then,

$$\begin{aligned} \frac{dr}{dt} &= \frac{1}{2}\left(\frac{1}{4}\right) = \frac{1}{8} \text{ in/yr } = \frac{1}{96} \text{ ft/yr} \\ \frac{dh}{dt} &= 1 \text{ ft/yr and the volume } V \text{ is } V = \pi r^2 h. \end{aligned}$$

Since r and h are functions of time, t,

$$\frac{dV}{dt} = 2\pi r h \frac{dr}{dt} + \pi r^2 \frac{dh}{dt}.$$

Thus, when $r = \frac{3}{2}$ ft. and $h = 50$ ft.,

$$\frac{dV}{dt} = 2\pi \left(\frac{3}{2}\right)(50)\left(\frac{1}{96}\right) + \pi \left(\frac{3}{2}\right)^2 (1) = \frac{61\pi}{16} \text{ ft}^3\text{/yr}.$$

Observation

The solution illustrates the *minimum* amount of information and explanation required for *full marks*.

4.1 General Guide for Solving Related Rate Problems

STEP 1. Draw a *large* diagram to illustrate the geometry of the problem at some arbitrary time t. Label any numerical quantities which remain fixed throughout the problem. Label all quantities which change with time by letters.

STEP 2. Find one or more relations among the quantities which vary. These relations must hold for all time. Usually the geometry or trigonometry of the diagram suggest an appropriate relation.

STEP 3. Differentiate these relations implicitly with respect to time.

STEP 4. Substitute in all given values such as rates and distances to enable you to solve the resulting equation for the required unknown.

The diagram should be large and neat so that you may see clearly the geometry of the situation. Use one third of a page or more for the diagram. If you have a *good* diagram to stare at your chances of completing the question are also good.

In problems involving ships, airplanes and autos, etc., travelling in different directions use a coordinate axis. If the object is moving to the right $D_t x$ is positive. If the object is moving to the left $D_t x$ is negative. If the object is moving upward in the diagram then $D_t y$ is positive. If the object is moving downward then $D_t y$ is negative.

The above choice is the most natural one. If you are consistent in its use you won't get things increasing when they are really decreasing or decreasing when they are supposed to be increasing.

4.2 Problems

1. A point moves on a hyperbola $y^2 - 2x^2 = 2$, where x and y are differentiable functions of time. What is the rate of change of y with respect to time when $D_t x = \frac{1}{2}$, $x = 1$, and $y = -2$?

   Step 1. Differentiate the equation of the hyperbola implicitly with respect to t.

   Step 2. Substitute in the given values for $D_t x$, x, and y then solve for $D_t y$.

2. Let $y = x^3$, where x is a differentiable function of t.

   (a) Suppose that when $x = 4$, $\frac{dx}{dt} = 3$. What is $\frac{dy}{dt}$ then?

   (b) Suppose that when $x = 3$, $\frac{dy}{dt} = 9$. What is $\frac{dx}{dt}$ then?

   (c) Suppose that when $y = 8$, $\frac{dx}{dt} = 2$. What is $\frac{dy}{dt}$ then?

3. Find $\frac{dy}{dt}$

   (a) $x^3 + y^3 - 3xy = 0$, $D_t x = -1$, $x = \frac{3}{2}$, $y = \frac{3}{2}$.

   (b) $b^2x^2 + a^2y^2 = a^2b^2$, $D_t x = \frac{1}{2}$, $x = \frac{a}{\sqrt{2}}$, $y > 0$.

   (c) $y^2 = x^2(x + 1)$, $D_t x = a$, $x = 1$, $y = -\sqrt{2}$.

   (d) $y = \frac{1}{2}\left(e^x + e^{-x}\right)$, $D_t x = 1$, $x = 1$.

4. Find $\frac{dy}{dt}$

   (a) $y = \ln x$, $D_t x = b$.

   (b) $y = \sin x$, $D_t x = 10$.

   (c) $y = A \sin x + B \cos x$, $D_t x = 1$, $x = \frac{\pi}{4}$.

   (d) $\cot y = \frac{x}{72}$, $\frac{dx}{dt} = -5$, $x = 36$.

   (e) $xe^{-x} - y = 0$, $\frac{dx}{dt} = e$, $x = 2$.

5. Find $\frac{dy}{dt}$

   (a) $y = Ce^x$, $\frac{dx}{dt} = K$, $x = 0$.

   (b) $y = Ce^x$, $\frac{dx}{dt} = \ln 2$, $y = 2C$.

   (c) $y = a \ln x + b$, $\frac{dx}{dt} = e^t$, $t = \ln x$.

   (d) $y = \sin x + 2e^y$, $D_t x = 4$, $x = 0$, $y = 0$.

6. A point moves along the curve $y = \cos x$, its x coordinate increasing at a steady rate of 1 unit per second. How fast is it rising when $x = \frac{\pi}{4}$? $x = \frac{\pi}{2}$? $x = \frac{3\pi}{2}$? Show that its vertical acceleration is proportional to its distance from the x-axis.

7. A point moves along the curve $y = a \cos x + b \sin x$ its x coordinate increasing at a constant rate of 1 unit per second. How fast is it rising when $x = \frac{\pi}{4}$? $x = \frac{\pi}{2}$? Show that its vertical acceleration is proportional to its distance from the x-axis.

8. A point moves along the curve $x - b = \ln y$ its x coordinate increasing at a constant rate of K units per second. How fast is it rising when $x = 0$? $x = b$? Show that its vertical velocity is proportional to its distance from the x-axis.

9. A point moves along the curve $y = \sqrt{x^2 + 1}$ in such a way that $\frac{dx}{dt} = 4$. Find $\frac{dy}{dt}$ when $x = 3$.

10. A point moves along the upper half of the curve $y^2 = 2x + 1$ in such a way that $\frac{dx}{dt} = \sqrt{2x + 1}$. Find $\frac{dy}{dt}$ when $x = 4$.

11. The variables x, y, and z are all functions of t and satisfying the relation $x^3 - 2xy + y^2 + 2xz - 2xz^2 + 3 = 0$. Find $\frac{dz}{dt}$ when $x = 1$, $y = 2$, if $\frac{dx}{dt} = 3$ and $\frac{dy}{dt} = 4$ for all times t.

12. Where on the ellipse $x^2 + 2y^2 = 6$ is $\frac{dx}{dt} = -\frac{dy}{dt}$ as the point (x, y) moves? [$(2, 1)$ or $(-2, -1)$]

13. On the parabola $y^2 = 2px$ where is $\frac{dx}{dt} = \frac{dy}{dt}$? What is the slope there?

14. Where on the graph of $y = x^3 + 3x^2 - 9x + 6$ is $D_t y = 0$, regardless of $\frac{dx}{dt}$.

15. Let $y = (x^2 + 1)^2$ and $u = (x^2 - 1)^2$, where x is a differentiable function of t. If $\frac{dy}{dt} = \frac{1}{4}$ when $x = 2$, what is $\frac{du}{dt}$?

16. Let $u = (x - 1)^3$ and $v = (x + 1)^3$, where x is a differentiable function of t. If $\frac{du}{dt} = 6$ when $\frac{dx}{dt} = \frac{1}{2}$, what is $\frac{dv}{dt}$ then?

17. If $y = u^2 - 2u + 1$ and $u = 2x^3 - 3x^2 + x - 5$, what is the rate of change of y with respect to x when $x = 2$? When $x = 1$?

18. If $w = u^4 + \left(\frac{1}{u}\right)$ and $u = \frac{1}{(1+v^2)}$, find the rate of change of w with respect to v when $v = 3$.

19. A particle moves in the plane so that at time t its coordinates are $x = t^2 + 1$, $y = \sqrt{2t^2 + 3}$. How fast is its distance from the origin changing when $t = 3$?

20. An electron moves in the plane so that at time t its coordinates are $x = a(e^t + e^{-t})$ and $y = b(e^t - e^{-t})$. How fast is its distance from the origin changing when $t = 3$? When $t = 0$?

21. An electron moves in the plane so that at time t its coordinates are $x = 3e^{\frac{1}{2}t}$, $y = 4e^{-\frac{1}{2}t}$. How fast is its distance from $(0, 0)$ changing when $t = 0$? When $t = 4$?

22. Water is flowing at the rate of 5 ft^3/min into a tank in the form of a cone of altitude 20 ft. and base radius 10 ft. and with its vertex in the downward direction. How fast is the water level rising when the water is 8 ft. deep?

    Step 1. Draw the diagram of the conical water tank (side view). Label the radius of the tank 10 and the altitude 20. Let the depth of the water in the tank at a given instant be h. Draw water level on the diagram. Let r be the radius of the surface of the water.

    Step 2. (a) Look at the geometry of the diagram. There are similar triangles. Using ratios of the sides find r as a function of h.
    (b) What is the volume of the cone of water as a function of r and h?
    (c) What is the volume of the cone of water as a function of h only.

    Step 3. Differentiate with respect to t.

    Step 4. Now substitute the value the question gives for $\frac{dV}{dt}$ and instantaneous value for h, and *then* solve for $\frac{dh}{dt}$.

23. A conical funnel is 8 in. across the top and 12 in. deep. A liquid is flowing in at a rate of 60 cu.in/sec and flowing out at a rate of 40 cu.in/sec. Find how fast the surface is rising when the liquid is 6 in. deep. Assume the funnel ends in a point for this problem.

24. Water is leaking out of a conical tank (vertex down) at the rate of 0.5 ft^3/min. The tank is 30 ft. across at the top and 10 ft. deep. If the water level is rising at the rate of $1\frac{1}{2}$ ft/min, at what rate is water being poured into the tank from the top?

    Step 1. Draw diagram of a conical water tank partly filled with water. Fixed dimensions of the tank should be labelled with numerical values. Variable dimensions of the water should be labelled with letters.

    Step 2. What is the volume of the cone of water. Use similar triangles to relate the dimensions of the water, (the height and the radius). Express the volume of the water as a function of the height of the water.

    Step 3. Note that the rate of change of the volume is the rate at which water pours in minus rate at which water leaks out. Now go ahead and differentiate and solve for the unknown quantity.

25. Water is flowing at the rate of 5 ft^3/min into a tank in the form of a cone of altitude 20 ft. and base radius 10 ft. and with its vertex in the downward direction. Find the rate at which the uncovered surface of the conical tank is decreasing when the water is 8 ft. deep.

    Step 1. To get the diagram, do it yourself, or, if you give up look at Step 1 of #22.

    Step 2. What is the surface area of a cone? Find the surface area of the dry part by subtracting the surface area of wet part from the whole cone. Finish it.

26. Water is flowing into a conical tank 12 ft. across and 12 ft. deep. If the water is rising at the rate of 1 in/min when the water is 6 ft. deep, what is the rate of flow?

    Step 1. Draw a diagram showing the side view. Do it yourself or, if you must, look back at Step 1 of #22. Label fixed dimensions with numbers and label variable dimensions with letters.

    Step 2. What is the volume of a cone of water? Express the volume of the cone of water as a function of the height. Finish it.

27. A student is using a straw to drink from a conical paper cup, whose axis is vertical, at the rate of 6 cubic in. a second. If the height of the cup is 10 in. and the diameter of its opening is 6 in., how fast is the level of the liquid falling when the cup is half full?

    Step 1. Draw a diagram of the cross-section of the cone. Label fixed dimensions with numbers and variable dimensions with letters.

    Step 2. What is the volume of a cone of liquid? What relationship can you see from the diagram between the dimensions? Write the volume of the cone of liquid as a function of the height of the liquid. Finish it. $[2\sqrt{4}/(3\pi) \approx .3369$ in/sec.]

28. Two automobiles start from a point A at the same time. One travels west at 80 mi/hr and the other travels north at 45 mi/hr. How fast is the distance between them increasing 3 hr later?

    Step 1. Draw a diagram. Let A be the origin. Let distance first auto goes be x, and the distance the second auto goes by y. Let s be the distance between the autos.

    Step 2. Using the diagram, express s^2 as a function of both x and y.

    Step 3. & 4. Differentiate and finish it.

29. At noon of a certain day, ship A is 60 mi. due north of ship B. If A sails east at 15 min/hr and B sails north at 12 mi/hr, determine how rapidly the distance between them is changing 2 hr. later. Is it increasing or decreasing?

    Step 1. Draw a diagram. Let A be at origin to start with. Label the distance A sails east x mi. Let B be on y axis below the origin. What are the coordinates of B at noon? Label the distance between A and B as s.

    Step 2. Express s^2 as a function of x and y, by using the geometry of the diagram.

    Step 3. Differentiate and solve.

30. One airplane flew over an airport at the rate of 300 mi/hr. Ten minutes later another airplane flew over the airport at 240 mi/hr. If the first airplane was flying west and the second flying south (both at the same altitude), determine the rate at which they were separating 20 minutes after the second plane flew over the airport.

    Step 1. Draw the diagram. Let the airport be at the origin 0. Let x be the distance the first plane flew and y be the distance the second plane flew. Let s represent the distance between the airplanes.

    Step 2. Using your diagram express s^2 as a function of the two variables x and y.

    Step 3. Differentiate s^2 with respect to t (t is for time).

    Step 4. Solve for $\frac{ds}{dt}$ which is the rate at which the planes are separating. Substitute in values for x min., y mi., $\frac{dx}{dt}$ mi/min and $\frac{dy}{dt}$ mi/min. (Watch out! Your were given speed of airplanes in mi/hr.) $\left[\frac{107}{17}\text{ mi/min}\right]$

31. An airplane, flying east at 400 miles per hour, goes over a certain town at 11:30 a.m. and a second plane, flying northeast at 500 miles per hour, goes over the same town at noon. How fast are they separating at 1:00 p.m.?

    Step 1. Draw a diagram using coordinate axes. Place the town at the origin. Assume planes flew at the same altitude. The triangle you get does *not* have a right angle in it.

    Step 2. If the distance between the two planes is called a, express a^2 as a function of the distances that the two planes flew. (It's called the law of cosines and is found in the trigonometry section of Chapter 0.) Finish it.

32. At noon ship A leaves port 0 steaming due south at 10 mi/hr. At 2 p.m. ship B leaves 0 going 60° east of south at 20 mi/hr. Find the rate at which the ships are separating at 5 p.m.

33. One ship leaves port and steams due north at 10 knots. Three hours later another ship leaves the same port and steams due west at 30 knots. How fast is the distance between them increasing when the first ship has been out of port for 5 hours?

34. A man starts walking eastward at 5 ft/sec from a point A. Ten minutes later a second man starts walking west at the rate of 5 ft/sec from a point B, 3000 ft. north of A. How fast are they separating 10 min. after the second man starts?

    Step 1. Draw a diagram. Join the point the first man is at (after about 20 minutes) to the point the second man is at. Make this line the hypotenuse of a right angled triangle. One side is the path of the first man produced as far west as second man has gone. The other side is a line 3,000 ft. north from the path of the first man (produced) to the position of the second man. Use as variable x, the distance each man has walked starting at the time the second man starts walking.

    Step 2. Using geometry find (hypotenuse)^2 as a function of x. Finish it. [$3\sqrt{10}$]

35. At a given instant the legs of a right triangle are 8 in. and 6 in., respectively. The first leg decreases at 1 in/min and the second increase at 2 in/min. At what rate is the area increasing after 2 min?

    Step 1. Draw a diagram. Orient it anyway you like but put the right angle between the 8 in. and 6 in. legs. Call the length of the legs x and y because they are varying with time.
    (The numbers 6 and 8 are instantaneous values — they are the ones you substitute in Step 2).

    Step 2. Express area as a function of both x and y. Finish it. [1 in^2/min.]

36. The base of a triangle is increasing at the rate of 4 in/min, while the altitude is decreasing at the same rate. At what rate is the area changing when (a) the base is 10 in. and the altitude 6 in.? (b) the base 6 in. and the altitude 10 in.

37. The sides of an equilateral triangle are increasing at the rate of 2 in/min. At what rate is the area increasing when the side is 5 in.?

38. The area of an equilateral triangle is increasing at the rate of 4 in^2/min. At what rate is one side increasing when the area is 10 in^2?

39. At a certain instant a small balloon is released (from ground level) at a point 75 ft. away from an observer (on ground level). If the balloon goes straight up at a rate of $2\frac{1}{2}$ ft/sec, how rapidly will it be receding from the observer 40 sec. later?

    Step 1. Draw a diagram. Let observer be at the origin. At a point 75 ft. away (measured along the x-axis) the balloon is released and starts straight up. Show it with a vertical line. Label the height y. Draw a line from the observer to the balloon and label the distance s.

    Step 2. Express s^2 as a function of y. Finish it.

40. An airplane at an altitude of 3,000 ft., flying horizontally at 300 mi/hr, passes directly over an observer. Find the rate at which it is approaching the observer when it is 5,000 ft. away.

    Step 1. Draw a diagram. Place the observer at the origin. Label a point directly above observer on y axis as $(0, 3000)$. If plane flies from left to right, then it is at a place to the left of the y-axis to start with at point with coordinates $(x, 3000)$. Label the distance from origin to plane as s.

    Step 2. Express s^2 as a function of x. Finish it.
    Comment — Rate at which plane approaches observe should turn out to be a negative number. (Because distance from observer to plane is *decreasing*.)

41. A boy $4\frac{1}{2}$ ft. tall walks toward a light 10 ft. above the ground at the rate of 6 fps. How fast is his shadow changing in length?

Step 1. Draw diagram. Label the heights of the lamp and the boy with the values given. Label the distance the boy is from the lamp x. Label the length of the boy's shadow as y. A line from the lamp touching the top of the boy's head is joined to the end of the shadow.

Step 2. Find a relationship between x and y by using the *similar triangles* in the diagram. The sides are in proportion. Finish it.

42. In problem #41 how fast is the shadow of his head moving?

Step 1. The diagram is the same as for #41. Let distance from the lamp to the boy be x, *but let the distance from the base of the lamp to the tip of the shadow's head by y.*

Step 2. Use similar triangles again to relate x and y. Finish it! [toward the light at $\frac{120}{11}$ fps.]

43. A man 6 ft. tall is walking horizontally at the rate of 84 ft. per minute directly toward a light which is 20 ft. above the ground. At what rate is the length of his shadow changing?

44. A lamp on a lamp post is 24 ft. high. A man of height 5 ft. 6 in. is walking directly away from the lamp post at the rate of 4 ft. per sec. Show that the length of the man's shadow increases at a constant rate. What is that constant rate?

45. A 6 ft. man walks away from a 12 ft. lamp post at the rate of 4 ft/sec. How fast is his shadow lengthening when he is 21 ft. from the post.

46. The shadow cast by a man standing 3 ft. from a lamp post is 4 ft. long. If the man is 6 ft. tall and walks await from the lamp post at a speed of 400 ft/min, at what rate will his shadow be lengthening:

    (a) a quarter of a minute later;
    (b) when he is 20 ft. from the lamp post?

47. A balloon is being inflated at the rate of 15 ft^3/min. At what rate is the diameter increasing after 5 min? Assume that the diameter is zero at time zero.

Step 1. Draw a diagram.

Step 2. What is the volume of a sphere? Express the volume of the sphere as a function of the diameter. Finish it.

48. A small spherical balloon is being inflated at the rate of 1 cu.ft/min. At what rate is the diameter increasing 2 sec. after inflation begins?

Step 1. Draw diagram. What is varying?

Step 2. What is the volume of a sphere? Finish it.

49. In problem number 48 how fast is the surface area of balloon increasing? [approx. 1.6 ft^2/min]

Step 1. Do problem number 48.

Step 2. What is the formula for the surface area of a sphere? Finish it.

50. Gas is pumped into a spherical balloon in such a way that the volume increases at the rate of 2 cu.ft/min. At what rate is the radius increasing when the radius is 12 ft.

51. Helium is pumped into a spherical balloon at the rate of 4 ft^3/min. At what rate is the radius increasing

    (a) when the radius is 3 ft.?
    (b) when the volume is 40 ft^3?

52. A snowball is melting at the rate of 2 cubic in. per min. If it remains spherical how fast is its surface area decreasing when its diameter is 16 in.?

Step 1. Draw the snowball. What dimension is varying.

Step 2. What is the volume of the snowball at an instant? What is the surface area of the snowball at the same instant?

53. As a spherical mothball evaporates, its volume decreases at a rate proportional to its surface area. Show that the rate of decrease of the radius is constant.

54. A spherical balloon is expanding in such a way that the radius is increasing at a rate proportional to the surface area. Show that the surface area is increasing at a rate proportional to the volume.

55. A spherical balloon is blown up so that its volume is V ft^3. If air is released at $t = 0$ at the rate of $1/(1+t^4)$ ft^3/sec, what is the rate of change of the radius of the balloon?

56. A 24 ft. ladder leans against a high wall. If the foot of the ladder is pulled away from the base of the wall at the rate of 6 ft/min, how fast is the top moving when the foot is 8 ft. from the base of the wall?

    Step 1. Draw a diagram using a coordinate axis with the origin at the base of the wall. Follow advice given in general guide to solving related rate problems concerning the direction parts are moving in. Label the distance of the bottom of the ladder from the wall as x, and the distance of the top of the ladder from the ground as y.

    Step 2. Express y^2 as a function of x. Finish it. (While you were not given the value of the y at the instant x is 8, you can calculate it. You need this at the very end of the question.)

57. A 10 ft. ladder is leaning against a vertical wall. If the bottom of the ladder is pulled away from the wall at the rate of 2 ft/sec, at what rate is the top of the ladder moving down the wall when the top is 8 ft. from the ground.

58. A 15 ft. ladder leans against a vertical wall. If the top slides downward at the rate of 2 ft/sec, find the speed of the lower end when it is 12 ft. from the wall.

59. Suppose that a 13 ft. ladder is leaning against a vertical wall with the foot of the ladder on a horizontal surface. The foot of the ladder is drawn away from the wall at the rate of 3 ft/sec. How fast is the top of the ladder moving down the wall when the foot of the ladder is 5 ft. from the wall?

60. A ladder 25 ft. long is leaning against a wall. If the lower end of the ladder is pulled away from the wall at the rate of 1 ft/sec, at what rate is the other end slipping down the wall when it is 24 ft. from the ground? $\left[\frac{41}{4\sqrt{34}}\text{ ft/sec}\right]$

61. A ladder 20 ft. long leans against the side of a house. The lower end slips away from the house at a rate of 3 ft/sec. When the lower end is 10 ft. from the wall, find the rates at which:

    (i) the other end slides downward;

    (ii) the ladder rotates.

62. At a given instant the legs of a right triangle are 5 cm. and 12 cm. long. If the short leg is increasing at the rate of 1 cm/sec and the long leg is decreasing at the rate of 2 cm/sec, how fast is the hypotenuse changing?

    Step 1. Draw diagram labelling all the lengths that very with a letter.

    Step 2. Express (hypotenuse)^2 as a function of the sides. Finish it.

63. One side of a right triangle is increasing at 2 units/sec, while the other side is decreasing in such a way that the hypotenuse remains constant at 10. When the length of the first side is 6 is the area increasing or decreasing, and at what rate?

64. A light is on the ground 48 ft. from a high wall. A man 6 ft. tall walks directly from the light toward the wall at the rate of 6 ft/sec. How fast is the top of his shadow moving down the wall when he is halfway there?

65. A light is on the ground 40 ft. from a building. A man 6 ft. tall walks from the light toward the building at 6 ft/sec. How rapidly is his shadow on the building growing shorter when he is 20 ft. from the building?

    Step 1. Draw a diagram. The light is fixed on the ground. Draw vertical lines for the man and the shadow on the building. Draw a straight line from the light on the ground to the top of the man's head to the top of the shadow on the building. Label the 40 ft. distance and the 6 ft. man. Label the height of the shadow as h, and the distance of the man from wall as x.

    Step 2. Use similar triangles to get a ratio which will relate h and x. Finish it.

66. A ball is thrown vertically upward and reaches a height of 100 ft. If the angle of elevation of the sun is 60 degrees, how fast is the shadow of the ball moving 2 sec. after it begins to fall? It falls at the rate of $-16t^2$ ft/sec. (t is time).

    Step 1. Draw a diagram. The angle of elevation is 60° means that the angle between the horizontal earth and the line joining shadow, ball and sun is 60°. The diagram is a triangle. Label the height y and the base x (x is the distance of the shadow from the point beneath the ball).

    Step 2. Use trigonometry. The angle 60° is a constant. Express x as a function of y using trigonometry. Finish it.

67. A rocket is fired vertically into the sky. An observer is located 4 miles from the launching pad. How fast is the distance between the observer and the rocket changing when the rocket has an altitude of 3 miles if it is ascending at the rate of 6 miles/sec? [$3\frac{3}{5}$ mi/sec]

68. A man on a pier pulls in a rope attached to a small boat at the rate of 1 fps. If his hands are 10 ft. above the place where the rope is attached, how fast is the boat approaching the pier when there is 20 ft. of rope out?

    Step 1. Draw diagram. You should get a triangle with a fixed altitude of 10 ft. (the height of the pier). The rope forms the hypotenuse and the distance the boat is along the surface of the water is the base. Choose letters from these two lengths because they are changing.

    Step 2. From the geometry of the triangle, write down a relationship between the hypotenuse and the horizontal distance of boat from pier. Finish it.

69. The dimensions of a box are 10 in., 12, and 16. If the shorter sides are decreasing at the rate of 0.2 in. per min., and the longest side is increasing at the rate of 0.3 in. per min., how fast is the volume changing? [decreasing at approximately 34 cu.in/min]

    Step 1. Draw a diagram of a box. Are any of the sides fixed? If so label them with a fixed number. Label all the sides that can change with a letter. (It is very important here to distinguish between a fixed dimension and a variable dimension with an *instantaneous* value.)

    Step 2. What's the volume of a box? Finish it. [decreasing at approx. 34 cu.in/min]

70. A trough 10 ft. long has a cross section that is an isosceles triangle 3 ft. deep and 8 ft. across. If water flows in at the rate of 2 cu.ft/min how fast is the surface rising when the water is 2 ft. deep?

    Step 1. Draw a large diagram of a trough, showing the wide triangular end. Its dimensions are fixed. Label them with the numerical values. Indicate the water partly filling the tank. Label the height of the water h, and the width of the water at its surface w.

70. Step 2. What is the volume of the water in terms of w and h. (Volume here is (area of an end) times (length of trough).) On the end of the trough you should see similar triangles. Use the similar triangles to find a ratio relating w and h. Now express volume of water as a function of h. Finish it.

71. A swimming pool is 25 ft. wide, 40 ft. long, 3 ft. deep at the shallow end, and 9 ft. deep at the deep end, the bottom being an inclined plane. If water is pumped into the pool at the rate of 10 ft^3/min, how fast is the water level rising when it is 4 ft. deep at the deep end?

   Step 1. Draw a diagram showing the side view of the tank partly filled with water which shows the inclined plane. The side view should look like a triangle. (This problem resembles the previous one.) Label the dimensions of the tank which are fixed. Label the dimensions of the water which are varying with letters.

   Step 2. What is the volume of the water in the tank when it is partly filled. (Area of side times width of tank.) The volume formula will be a function of two variables. Use similar triangles seen in the side view of the tank to relate the two variables. Now find the volume of the water in the tank as a function of one variable only. Finish it.

72. A baseball diamond is 90 ft. on a side. (It is really a square.) A man runs from first base to second base at 25 ft/sec. At what rate is his distance from third base decreasing when he is 30 ft. from first base? At what rate is his distance from home plate increasing at the same instant?

   Step 1. Draw a diagram. A square 90 ft. on a side will do. The man going from first to second base should have a line drawn from him to third base. Thus a triangle is formed with the man, second base and third base at the vertices. Let distance man is from second base be x. Let distance man is from third base be s.

   Step 2. Express s^2 as a function of x. Finish it. Do second part of question after drawing a line from runner to home plate.

73. A baseball diamond is a 90 ft. square. A ball is batted along the third-base line at a constant speed of 100 feet per second. How fast is its distance from first base changing when

   (a) it is halfway to third base? [$20\sqrt{5}$ ft/sec]

   (b) it reaches third base? [$50\sqrt{2}$ ft/sec]

74. Sand is issuing from a spout at the rate of 3 ft^3/min and falling on a conical pile whose diameter at the base is always three times the altitude. At what rate is the altitude increasing when the altitude is 4 ft?

   Step 1. Draw the conical pile. Label its variable dimensions, such as the height and the radius of the base with letters.

   Step 2. What is the formula for the volume of a cone? What is the relationship between the diameter of the base and the height? Express the volume of the cone as a function of the height. Finish it. $\left[\frac{1}{12\pi}\text{ ft/min}\right]$

75. A pile of sand being dumped forms a right circular cone in which the altitude is $\frac{2}{3}$ the diameter. If the sand is dumped at 3 ft^3/sec, find the rate of increase of the diameter of the pile when it is 6 ft. high.

76. A plane flying at 1 mi. altitude is 2 mi. distant from an observer, measured along the ground, and flying directly away from the observer at 400 mph. How fast is the angle of elevation changing?

   Step 1. Draw a diagram. Label the angle of elevation as θ and the distance the plane has flown (measured along the ground) as x. Label the altitude as 1 mi.

   Step 2. Using trigonometry relate the x to the θ. Finish it.

77. A satellite shortly after launch is 50 mi. high and 25 mi. down range. If it is travelling at 2 mi/sec at an angle of 30 deg. with horizontal, how fast is the angle of elevation at the launch site changing?

[decreasing approx. 1.13 deg/sec]

Step 1. Use a coordinate axis with the origin 0 as the point of launching. Shortly after launch the satellite is 50 mi. high and 25 mi. down range. Draw this and label the position at this instant as A with coordinates $(25, 50)$. From point A the satellite travels at an angle of 30 deg. with the horizontal. Draw a line from A at 30 degrees with the horizontal. Label the length of the new line as s, and call some point on the new line B. Join B to the origin. Label as θ, the angle between the x-axis and OB.

Step 2. What are the coordinates at point B? (That is (x, y) at B are found by saying
$x = 25+$ (a function of s),
$y = 50+$ (a function of s).)
Next relate θ with x and y using trigonometry. Finally relate θ with a function of s.

Step 3. Differentiate with respect to t and solve. [decreasing at approx. 1.13 deg/sec]

78. A cylindrical reservoir is 10 ft. high and 10 ft. in diameter. Water flows in at the rate of $1\frac{1}{2}$ cu.ft/sec. How fast is the water level rising?

Step 1. Draw a diagram. The axis of the cylindrical reservoir is vertical. Label dimensions.

Step 2. What is the volume of a cylinder of water? Finish it.

79. A vat is in the shape of a rectangular parallelepiped, with a horizontal base having the dimensions 4 ft. by 6 ft. Oil is flowing into the vat at the rate of 30 cubic feet per minute. How fast is the oil level rising?

80. A trough that is 12 ft. long and 2 ft. high is 2 ft. wide at the top and has triangular ends. If water is put in at the rate of 1 ft^3/min, how fast is the depth increasing when it is 1.5 ft. deep.

81. A cylindrical tank with axis horizontal has a diameter of 6 ft. and a length of 15 ft. It holds oil to a depth of 4 ft. when a leak starts to drain off the oil at the rate of 10 cu.ft/min. How fast is the level falling?

Step 1. Draw a diagram showing the end view of the cylindrical tank lying on its side. When the tank leaks what dimensions are variable. Label these dimensions with letters.

Step 2. First of all you need an expression for the volume of the oil. Look up the formula for a circular segment. Looking at the end of the tank, the area covered by oil is the area of a circle minus the circular segment at the top where there is an air gap. Volume of oil is (area of end times length of the tank.) Next mark θ on the diagram. (The formula for the area of a circular segment involves an angle θ.) Using trigonometry note a relationship between your variable height dimension and θ. (Say $\cos\theta = ?$, or $\tan\theta = ?$) Also $(\text{radius})^2 =$ some function of variable dimensions.

Step 3. Differentiate with respect to t "the volume of the oil expressed in terms of θ."
Differentiate with respect to t (implicitly) "the relation between your variable height dimension and θ."

Step 4. You now have two relations differentiated with respect to t. Both results will have $D_t\theta$ in it somewhere. Eliminate the $D_t\theta$ by substituting an expression for it from one equation into the other equation. Hopefully you now have a single equation involving a D_tVol. and a height dimension. You may now substitute the numerical values into the equation and solve for the rate of change of height.

Comment: In this solution two relations were differentiated because it would be very awkward to find a single expression relating the height and the volume. It would have meant finding an expression for θ as a function of height. It is *usually* recommended that you find one expression in terms of one variable before differentiation. This problem illustrates a convenient alternative.

82. Water flows into a hemispherical tank of radius 10 feet (flat side up). At any instant, let h denote the depth of the water, measured from the bottom, r the radius of the surface of the water, and V the volume of the water in the tank. Computer $\frac{dV}{dh}$ at the instant when $h = 5$ feet. If the water flows in at a constant rate of $5\sqrt{3}$ cubic feet per second, computer $\frac{dr}{dt}$, the rate at which r is changing, at the instant t when $h = 5$ feet.

83. A hemispherical bowl is 2 ft. in diameter. Liquid is poured in at the rate of 200 cu.in/min.

    How fast is the surface rising just as it overflows? [approx. 0.44 in/min]

84. Water flows out of a spherical reservoir 20 ft. in diameter. If the water level is falling at the rate of 1 in/min when the water is 15 ft. deep, how fast is the volume decreasing?

85. A long level highway bridge passes over a railroad track which is 100 feet below it and at right angles to it. If an automobile travelling 45 mi. per hour is directly above a train going 60 miles per hour, how fast will they be separating 10 seconds later?

    Step 1. Draw a diagram on coordinate axis. Fix the origin somewhere.

    Step 2. Express (dist. between automobile and train)^2 as a function of the distances they both have gone. Finish it.

86. A lineman climbs a telephone pole at the rate of 2.5 feet per second while his boss sits in the shade of a neighbouring tree watching him. If the ground is level and the boss is 36 feet from the base of the pole, how many seconds must the lineman climb for the distance between him and the boss to be increasing at the rate of 1 foot per second?

    Step 1. Draw a diagram. It should show a right-angled triangle from the boss to the base of the pole, up the pole to the lineman and back to the boss. Label the height the lineman has climbed y and the distance from the lineman to the boss as s.

    Step 2. Express s^2 as a function of y. Finish it.

87. A sixteen-foot length of metal pipe is leaning against a wall. If the bottom of the pipe is pulled along the level pavement, directly away from the wall, at 2 feet per second how fast is the height of the midpoint of the pipe decreasing when the foot of the pipe is 4 ft. from the wall?

    Step 1. Draw a diagram. What distance is fixed? Label it. What varies. Use letters for the items that change. It would be a good idea to put it on coordinate axes with the origin at the base of the wall, the foot at $(x, 0)$ and the top of the ladder at $(0, y)$.

    Step 2. Relate the x and y using Pythogoras' theorem. Use similar triangles to relate the height of the midpoint to y. Finish it.

88. Two street lights, each 60 feet high, are 100 feet apart. The light at the top of one is functioning but the other is being repaired by a workman. If the workman drops his tool kit from the top of the second pole, how fast is its shadow moving when the kit is 20 ft. from the ground?

    Step 1. Draw a diagram. The tool kit falls. A straight line is drawn from the top of the lighted pole to the tool kit to the ground. Label the distance the tool kit is from the top of the pole where it was dropped as s. (The height the tool kit is above the ground is $(100 - s)$. Label the distance the top of the shadow is to the second pole as x.

    Step 2. Use geometry to relate the variable s to the variable x. Similar triangles should be obvious.

    Step 3. Differentiate the relation in Step 2 with respect to t.

    Step 4. The velocity of a falling object is $v = \sqrt{2as}$ where a is acceleration due to gravity in ft/sec^2. s is distance it fell. From this you get $D_t s$.

89. A pebble thrown into a still pond produces concentric circular ripples. If the radius of the largest ripple increases at 2 ft/sec, how fast is the area of disturbance within the ripple increasing when the radius is 15 feet?

90. A stone is dropped into a still pond. Concentric circular ripples spread out, the radius of the disturbed region increasing at the rate of a ft/sec. At what rate does the area of the disturbed region increase when the radius of that region is already b ft.

91. A stone dropped in a still pond produces a circular ripple.

    (a) If the radius increases at 2 ft/sec, how fast is the enclosed area increasing when the radius is 12 ft.

    (b) If the area increases at 24 sq.ft/sec, how fast is the radius increasing when the radius is 12 ft.

92. A stone thrown into a pond produces a circular ripple which expands from the point of impact. If the radius of the ripple increases at the rate of 1.5 ft/sec, how fast is the area growing when the radius is 8 ft?

93. Two concentric circles are expanding, the outer radius at the rate of 2 ft/sec and the inner one at 5 ft/sec. At a certain instant the outer radius is 10 ft. and the inner radius is 3 ft. At this instant, is the area of the ring between the two circles increasing or decreasing? How fast?

94. The radius of a right-circular cylinder increases at constant rate. Its altitude is a linear function of the radius and increases three times as fast as the radius. When the radius is 1 ft. the altitude is 6 feet. When the radius is 6 ft, the volume is increasing at a rate of 1 cubic ft. per second. When the radius is 36 ft, the volume is increasing at a rate of n cubic ft. per second, where n is an integer. Compute n.

95. A cylindrical metal cylinder is expanding due to heating with the base radius increasing at the rate of $.002''$ and the altitude increasing at the rate of $.001''$ per minute. At an instant when the radius is $10''$ and the altitude is $60''$, find the rates of change per minute of the volume and the curved surface area of the cylinder.

96. A cylindrical steel ingot is shrinking in cooling with the base radius decreasing at $.001''$ per minute and the altitude decreasing at $.005''$ per minute. At an instant when the radius is $20''$ and the altitude is $100''$, find the rate of decrease in the volume of the ingot. [decreasing at 6π cu.in/min]

97. The volume of a cube is increasing at the rate of 7 cubic in. per minute. How fast is the surface area increasing when the length of an edge is 12 in?

    Step 1. Draw a diagram of a cube labelling anything variable.

    Step 2. What is the formula for the volume of a cube? Finish it.

98. Each edge of a cube is expanding at the rate of 1 centimetre (cm) per second. How fast is the volume changing when the length of each edge is (a) 5 cm? (b) 10 cm? (c) x cm?

99. The volume of a pyramid is increasing at the rate of 30 cubic inches per second and the area of the base is increasing at the rate of 5 square inches per second. When the area of the base is 100 sq.in. and the altitude is 8 in. how fast is the altitude increasing?

100. A rolling snowball picks up new snow at a rate proportional to its surface area. Assuming that the snowball always remains spherical show that its radius is increasing at a constant rate.

101. Assume that water condenses on the surface of a spherical drop of water in such a way that the volume increases at a rate equal to k times the surface area. At what rate does the radius and surface area of the drop increase?

102. A circular wheel of radius 5 feet lies over the xy-plane, with its center at the origin, and revolves steadily in a counterclockwise direction at a rate of 25 revolutions per minute. How fast is the y coordinate of a particle on the wheel increasing as the particle goes through the point $(3, 4)$? The same question for its x coordinate.

103. Referring to the above problem, suppose that the wheel is rotating at a constant rate of one radian per minute. Show that then $\frac{dy}{dt} = x$ for all positions of the particle.

104. Suppose a particle traces a circle about the origin in such a way that $\frac{dy}{dt} = x$. Show that the particle is moving along the circle at the constant rate of 1 radian per unit of time.

105. A lighthouse is $\frac{1}{2}$ mi. out from a long straight line of cliffs. If the lighthouse beam rotates at the rate of 3 rpm, how fast is the light spot on the cliff moving at a point 1 mi. along the shore from the point nearest the lighthouse?

106. A lighthouse stands 100 feet from a straight shore. The light revolves at a rate of one complete revolution every 20 seconds. As it revolves it casts a spot of light on the shore. How fast is the spot of light moving along the shore when it is

    (a) at the point nearest the lighthouse;

    (b) 100 feet away from this point?

107. A variable right triangle ABC in the xy-plane has its right angle at vertex B, a fixed vertex A at the origin, and the third vertex C restricted to lie on the parabola $y = 1 + \frac{7}{36}x^2$. The point B starts at the point $(0, 1)$ at time $= 0$ and moves upward along the y-axis at a constant velocity of 2 cm/sec. How fast is the area of the triangle increasing when $t = \frac{7}{2}$ sec?

108. A point moves on the graph of $y = \sqrt{x}$. At which point are its x-coordinate and y-coordinate changing at the same rate?

109. A cone has a base of radius r and altitude h. If r is increasing at the rate of $\frac{2}{3}$ in/sec and h is decreasing at the rate of -1 in/sec, at what rate is the volume of the cone changing when $r = 4$ and $h = 3$?

110. A particle P is moving in an xy-plane. At any instant t, the x-coordinate of P is increasing at the rate of .2 unit per second, and the y-coordinate is decreasing at the rate of .3 unit per second. At what time-rate is the distance of P from the origin changing at the instant when P is at $(7, 24)$?

    [decreasing at .232 units/sec]

111. the combined electrical resistance, R, resulting from two resistances R_1 and R_2, in ohms, connected in parallel, satisfies:
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$
If R_1 and R_2 are increasing at the rates .5 and .4 per second, respectively, find the rate at which R is changing when $R_1 = 100$ and $R_2 = 200$.

112. An airplane flies in level flight at constant velocity, 8 mi. above the ground. (In this exercise assume the earth is flat). The flight path passes directly over a point P on the ground. The distance from the plane is decreasing at the rate of 4 miles per minute at the instant when this distance is 10 miles. Compute the velocity of the plane in miles per hour.

113. A boat sails parallel to a straight beach at a constant speed of 12 miles per hour, staying 4 miles offshore. How fast is it approaching a lighthouse on the shoreline at the instant it is exactly 5 miles from the lighthouse?

114. A boat is anchored in such a way that its deck is 25 ft. above the level of the anchor. If the boat drifts directly away from the point above the anchor at the rate of 5 ft/min how fast does the anchor rope slip over the edge of the deck when there are 65 ft. of rope out? (Assume that the rope forms a straight line from deck to anchor.)

115. A walk is perpendicular to a long wall, and a man strolls along it away from the wall at the rate of 3 fps. There is a light 8 ft. from the walk and 24 ft. from the wall. How fast is his shadow moving along the wall when he is 20 ft. from the wall?

116. A baseball player 15 ft. off first base tries to steal second. His running speed is 22 ft/sec, and the bases are 90 ft. apart. After a 2.5 sec. delay, the catcher throws the ball at 90 mph to the second baseman. At what rate does the ball approach the runner? Is the steal successful?

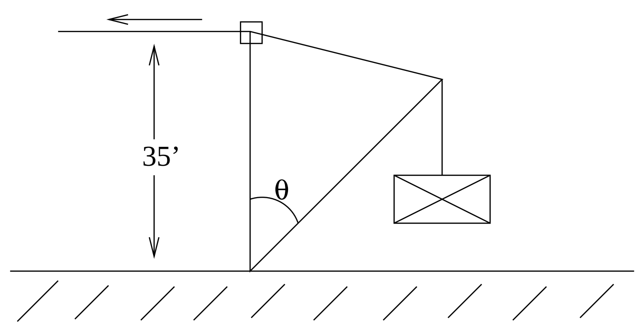
117. A ball falls from a height of 80 ft. How fast is the angle of elevation changing for an observer whose eye is 6 ft. above ground level, and who is 60 ft. from the point below the ball, when the ball is 26 ft. above ground. If s is distance ball has fallen, $D_t s = \sqrt{64s}$.

118. A bridge is 30 ft. above a canal. A motor boat going 10 ft/sec passes under the center of the bridge at the same instant that a man walking 5 ft/sec reaches that point. How rapidly are they separating 3 sec. later?

119. In problem 118, the man reaches the center of the bridge 5 sec. before the boat passes under it. Find the rate at which the distance between them is changing 3 sec. after the man crosses. $\left[\frac{125}{\sqrt{181}} \text{ ft/sec}\right]$

120. A tanker breaks in half spilling 10,000 cu.ft. of oil all at once. The oil floats on the surface and spreads. It is approximately a cylinder in shape and is becoming thinner and thinner. The uniform thickness, h, of the oil slick is given by $h = kt^{-\frac{1}{2}}$. (Where t is time and $k = 10^{-4}$ ft/hr$^{\frac{1}{2}}$ is a constant). Find the radius of the slick and it's rate of spread 4 hrs. later. $\left[\sqrt{\frac{2}{\pi}}10^4 \text{ ft; } \frac{10^4}{16}\sqrt{\frac{2}{\pi}} \text{ ft/hr}\right]$

121. The boom of a derrick is 30 ft. long. A cable attached to the boom ranges over a pulley at the top of a 35 ft. pole as shown. If the cable is drawn at the rate of 2 ft/sec to lift the weight, how fast is the angle between the boom and the pole changing when $\theta = 12°$? How fast is the weight being lifted at this instant?



122. A bacterial culture growing on a food medium in a shallow dish will generally have the shape of a circular disk whose area is proportional to the number of organisms. Suppose that at noon of one day such a culture has a diameter of 1 millimetre. If the population, P, of the bacteria is given by $P(t) = P(0)e^{.02t}$ (t measured in hours). Find the rate at which the diameter of the culture should be changing at 3 p.m. of the same day.

4.3 Economic Applications

If $C = f(x)$ is a function that relates cost, C, to the number of items sold, x, it may also be true that both x and C are functions of time. Hence we may obtain information about the rate of change of cost with time or the rate of sales per unit time depending on what information is known.

Example. The cost in dollars of selling x items is given by $C = 500 + x + \frac{1}{x}$. When the 50th item is sold, it is noted that the rate of sales is 20 items per hour. What is the rate of change of the cost with respect to time at that moment?

Solution. We are given $\frac{dx}{dt} = 20$ and $x = 50$ and are asked to find $\frac{dC}{dt}$. C and x are both implicit functions of time. Therefore, we use the chain rule to take the derivative with respect to time of both sides of $C = 500 + x + \frac{1}{x}$.

$$\begin{aligned} \frac{dC}{dt} &= 0 + \frac{dx}{dt} + \left(-\frac{1}{x^2}\right)\frac{dx}{dt} \\ &= \frac{dx}{dt} - \frac{1}{x^2}\frac{dx}{dt} = \left(1 - \frac{1}{x^2}\right)\frac{dx}{dt}. \end{aligned}$$

Substituting 50 for x and 20 for $\frac{dx}{dt}$ gives

$$\begin{aligned} \frac{dC}{dt} &= \left(1 - \frac{1}{50^2}\right)20 = \left(1 - \frac{1}{2500}\right)20 = \frac{2499}{2500}(20) \\ &= \frac{2499}{125} \approx \$20/\text{hr}. \end{aligned}$$

This means that the cost at the time the 50th item is sold is increasing at the rate of about \$20 per hour.

In this example, the cost per hour is a function of both the number of items sold and the rate of sales. If the rate of sales is 5 items per hour at the time the second item is sold, then the cost per hour is only \$3.75, which is computed as follows.

$$x = 2, \quad \frac{dx}{dt} = 5, \quad \text{and} \quad \frac{dC}{dt} = \left(1 - \frac{1}{x^2}\right)\frac{dx}{dt},$$

so

$$\frac{dC}{dt} = \left(1 - \frac{1}{4}\right)5 = \frac{3}{4}(5) = 3.75.$$

The cost per hour is a function of the number of items sold as well as the rate of sales because an increase in the number of items sold causes greater production costs, and an increase in the rate of sales causes greater sales cost due to a need for more salesmen, delivery personnel, and so forth.

123. The cost of selling x items in dollars is given by $C = 0.75x + 1,000$. What is the rate of change in the cost with respect to time at the instant that the 100th item is sold if the items are selling at the rate of 4 per hour?

124. Find the rate of change in cost, when the cost in dollars is given by $C = 700 + 8\sqrt{x}$ and the 36th item is sold, if the items are selling at the rate of 50 per hour.

125. Find the rate of change per week in cost, when the cost in dollars is given by $C = 100 + 2x + \frac{1}{x^2}$ and the first item is sold, if the items are selling at 1 a week.

126. Find the rate of change of cost, if the cost in dollars is given by $C = 800 + 9x^{\frac{2}{3}}$, when the 8th item is sold, and the items are selling at the rate of 100 per day.

127. Find the rate of change in cost, where cost is given by $C = 0.0002x^3 - 0.05x^2 + 20x + 20000$, when the 1000th item is sold, if the rate of sales is 50 per day. [26,000]

128. What is the rate of change in profit, given by $P = 4x^2 + 5x - 1000$, at the time the 100th item is sold, given that the items are selling at the rate of 70 a day?

129. The profit, P, resulting from the sales of x data-wacks is given by $P = x^2 + 500x - 100$, and exactly two data-wacks are sold every day.

(a) How is the profit changing when the 100th data-wack is sold?

(b) How is the profit changing when the 125th data-wack is sold?

(c) How is the profit changing when the 150th data-wack is sold?

130. A corporation, M, can produce x thousands of electric shavers in a year at a cost C, where

$$C = x^3 - 12x^2 + 96x + 30,$$

and the unit for money is \$100. When $x = 6$ the annual output is increasing at the rate of fifty shavers per month. Find the rate per month at which the annual cost is changing.

131. In the manufacture of electric washing machines, a corporation finds that the proper selling price, H, in dollars per machine to assure a desired margin of profit is given by $H = 3x^2 + 2x + 7$, where x is a cost index number associated with the business. Find the rate of change of H at an instant when $x = 9$ and x is increasing at the rate of .02 per month.

132. The production equation for a manufacturer, M, of air conditioners is $z = 2x^2 + 5xy + 3y^2$ where z trade units of the product are produced when M uses x units of the production factor labour and y units of the factor of land (which may be thought of as representing invested capital). Find the rate at which z is changing if x is decreasing at the rate of .03 unit per year and y is increasing at the rate of .5 unit per year when $x = 8$ and $y = 12$.

133. The demand, z wholesale units, for chickens in a certain market area is given by $z = 5800 - 6x^2 + 5y^2$ where the price in cents per pound of chickens is x and of turkeys is y. If x is decreasing at the rate of \$.005 per week, and y is increasing at the rate of \$.004 per week, find the rate at which z is changing when $x = 25$ and $y = 30$.